

POWER ELECTRONICS

First Revision :

Transient :

RL NATURAL RESPONSE

$$\tau = \text{time constant} = \frac{L}{R}, \quad i(t) = I_0 e^{-(R/L)t}, \quad t \geq 0,$$

$$v(t) = I_0 R e^{-t/\tau}, \quad t \geq 0^+, \quad p = I_0^2 R e^{-2(R/L)t}, \quad t \geq 0^+.$$

$$w = \int_0^t p dx = \int_0^t I_0^2 R e^{-2(R/L)x} dx = \frac{1}{2} L I_0^2 (1 - e^{-2(R/L)t}), \quad t \geq 0.$$

RC NATURAL RESPONSE :

$$\tau = RC.$$

$$v(t) = V_0 e^{-t/\tau}, \quad t \geq 0,$$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, \quad t \geq 0^+,$$

$$p = vi = \frac{V_0^2}{R} e^{-2t/\tau}, \quad t \geq 0^+,$$

$$\begin{aligned} w &= \int_0^t p dx = \int_0^t \frac{V_0^2}{R} e^{-2x/\tau} dx \\ &= \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), \quad t \geq 0. \end{aligned}$$

Step Response of RL

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-(R/L)t}.$$

$$v = L \left(\frac{-R}{L} \right) \left(I_0 - \frac{V_s}{R} \right) e^{-(R/L)t} = (V_s - I_0 R) e^{-(R/L)t}.$$

Step Response of RC circuits

$$v_C = I_s R + (V_0 - I_s R) e^{-t/RC}, \quad t \geq 0.$$

General solution :

$$x(t) = x_f + [x(t_0) - x_f] e^{-(t-t_0)/\tau}.$$

Sequential Switching

كل الفكره ه رسم الداييره في كل حاله وادرس تاثير كل واحده فيهم على العناصر

. CH2 :

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}.$$

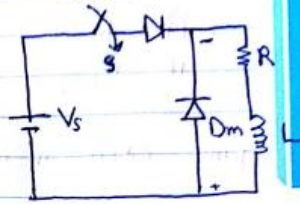
Where

Parameter	Terminology	Value In Natural Response
s_1, s_2	Characteristic roots	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
α	Neper frequency	$\alpha = \frac{1}{2RC}$
ω_0	Resonant radian frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$

	S_1 & S_2	Voltage is
$\omega_0^2 = \alpha_0^2$	Real & Equal	Critically Damped
$\omega_0^2 < \alpha_0^2$	Real & Distinct	Over Damped
$\omega_0^2 > \alpha_0^2$	Complex & Conj.	Under Damped

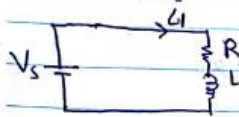
* Free Wheeling diode with R.L Load :-

اذا تم اغلاق المفتاح في السيار يمر التيار الى Load . ويتسبب وجود الملف فالكثبان (عنصر سعة) وتخزينه للطاقة فلا يتم فتح المفتاح فيحصل Spark ولذا يتم وضع F.O يتم تفريغ السيار في هذا الحسار



After closing switch

mode (1)

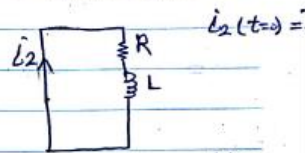


$$i_1(t) = \frac{V_s}{R} (1 - e^{-t/\tau})$$

When switch opened
at $t = t_1$

$\therefore I$ is Const = I_1

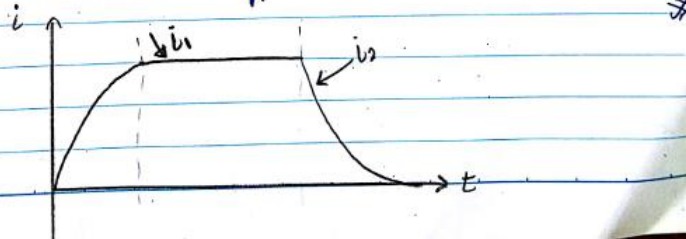
mode (2)



$$i_2(t) = I_1 e^{-t/\tau}$$

$$I_1 = \frac{V_s}{R} (1 - e^{-t_1/\tau})$$

$$\therefore i(t) = \frac{V_s}{R} (1 - e^{-t/\tau}) + I_1 e^{-t/\tau}$$



Laplace :

Ch1 : introduction

Reverse recovery time :

$$t_{rr} = t_a(\text{from charges}) + t_b(\text{from semiconductor})$$

$$I_{RR} = t_a \frac{di}{dt} \rightarrow (1)$$

$$Q_{RR} = \frac{1}{2} I_{RR} t_{rr}$$

$$t_{rr} = \sqrt{\frac{2Q_{RR}}{\frac{di}{dt}}}$$

$$I_{RR} = \sqrt{2Q_{RR} \frac{di}{dt}}$$

$$I_{av} = \frac{1}{T} \int_0^T i(t) dt$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

Thyristors Types

The most popular and much used thyristor types are shown in (Fig. 2.28):

- 1- Phase controlled thyristor (SCR)
- 2- Fast switching thyristor (SCR)
- 3- Gate-turn-off thyristor (GTO)
- 4-Bidirectional triode thyristor (TRIAC)
- 5-Light activated silicon-controlled rectifier (LASCR)

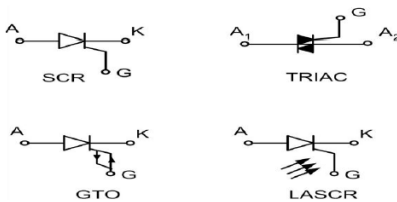


Fig. 2.28 Different Thyristor types symbols

Protection of SCR from di/dt and dv/dt

It is known that the coil current is not suddenly changed, so that a coil is connected in series with thyristors to limit the sudden change of di/dt.

Also, it is known that the capacitor voltage is not suddenly changed so that a series RC circuit, snubber circuit, is connected in parallel with thyristors to be protected from high dv/dt.

Fig. 2.32 illustrated the SCR circuit protected from di/dt and dv/dt.

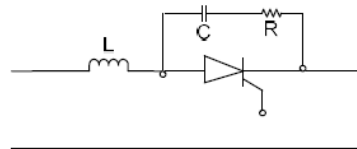


Fig. 2.32 SCR protected circuit from di/dt and dv/dt

Self-Commutation

This occurred when the anode current is reduced from the circuit itself under the holding current level.

Forced-Commutation

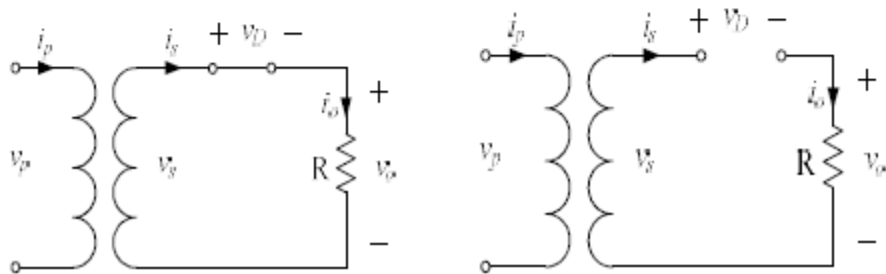
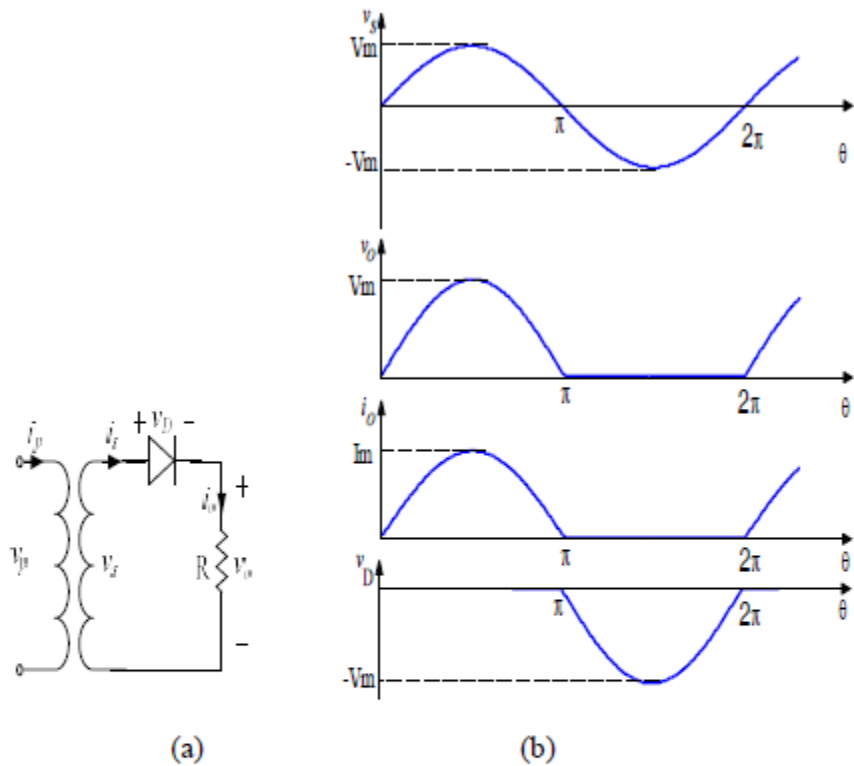
This occurred when the anode current is reduced under the holding current level using external effect. There are three methods of forced commutation are:

- Series switch.
- Parallel switch
- Applying reversed voltage across thyristor terminal.

CH3 : Uncontrolled Rectifiers

- Single phase - three phase

1- Single Phase half-wave rectifier (R Load)



$$V_{dc} = \frac{1}{T} \int_0^T v_o d\theta = \frac{1}{2\pi} \int_0^\pi V_m \sin\theta d\theta = \frac{V_m}{2\pi} [-\cos\theta]_0^\pi = \frac{V_m}{2\pi} [-\cos\pi + \cos 0] = \frac{V_m}{\pi}$$

o/p volt. avg

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}$$

output current average

$$I_A = \frac{1}{T} \int_0^T i_1 d\theta = \frac{1}{2\pi} \int_0^\pi \frac{V_m}{R} \sin\theta d\theta = \frac{V_m}{\pi R} = I_{dc}$$

average diode current

$$P_{dc} = I_{dc} V_{dc} = \frac{V_m^2}{\pi^2 R} \quad ((\text{Average output power}))$$

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{T} \int_0^T v_o^2 d\theta} = \sqrt{\frac{1}{2\pi} \int_0^\pi V_m^2 \sin^2 \theta d\theta} = \sqrt{\frac{V_m^2}{2\pi} \int_0^\pi \frac{1}{2} (1 - \cos 2\theta) d\theta} \\ &= \sqrt{\frac{V_m^2}{4\pi} [\theta - \frac{1}{2} \sin 2\theta]_0^\pi} = \frac{V_m}{2} \sqrt{\frac{1}{\pi} [\pi - 0 - \frac{1}{2} \sin 2\pi + \frac{1}{2} \sin 0]} \\ &= \frac{V_m}{2} \sqrt{\frac{1}{\pi} [\pi - 0]} = \frac{V_m}{2} \end{aligned}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R}$$

$$I_R = \sqrt{\frac{1}{T} \int_0^T i_1^2 d\theta} = \sqrt{\frac{1}{2\pi} \int_0^\pi \frac{V_m^2}{R^2} \sin^2 \theta d\theta} = \frac{V_m}{2R} = I_{rms}$$

diode rms current

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m^2}{4R}$$

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\left(\frac{V_m^2}{\pi^2 R}\right)}{\left(\frac{V_m^2}{4R}\right)} \times 100 = \frac{4}{\pi^2} \times 100 = 40.53\%$$

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} = \sqrt{\left[\frac{\left[\frac{V_m}{2}\right]}{\left[\frac{V_m}{\pi}\right]}\right]^2 - 1} = \sqrt{\frac{\pi^2}{4} - 1} = 1.2114$$

this means that the amount of AC

component present at the output is 121.14% of the DC component

Transform Utilization Factor TUF

$$TUF = \frac{P_{dc}}{S} = \frac{\frac{V_m^2}{\pi^2 R}}{\frac{V_m^2}{2\sqrt{2} R}} = \frac{2\sqrt{2}}{\pi^2} = 0.2867$$

Means from every 1000 VA from source we utilize 286.7 watt

Q: What are the disadvantages of single-phase half-wave rectifier

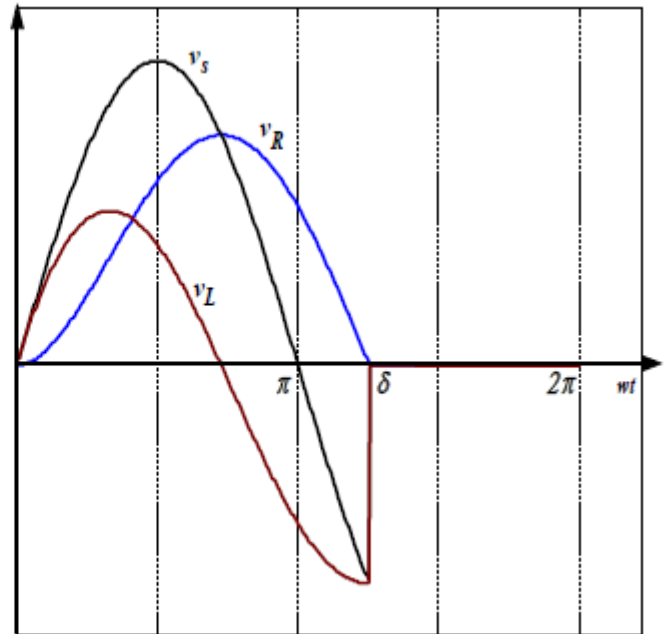
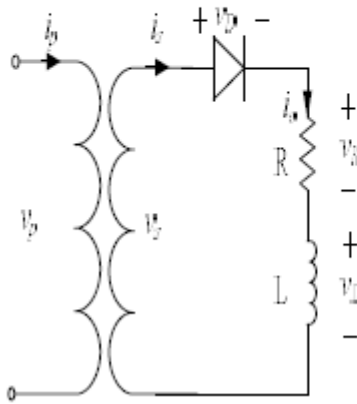
The disadvantages of single-phase half-wave rectifier are:

1. The rectifier efficiency is very low only 40.53 %.
2. The ripple factor is very high which means a large value of AC component

compared to the DC component in the load voltage.

3. Low transformer utilization factor, because of DC saturation of the transformer secondary. Due to the current flows only on positive half cycle, therefore, this unidirectional current saturates the secondary winding of the transformer.

2- Single Phase half-wave rectifier (R-L Load)(without wheeling diode)



Complementary solution : $wL \frac{di}{d\theta} + Ri = 0 \rightarrow i(\theta) = Ae^{\frac{R}{wL}\theta}$

steady state solution : $i(\theta) = \frac{V_m}{Z} \sin(wt - \phi)$

$$i(\theta) = Ae^{\frac{R\theta}{wL}} + \frac{V_m}{Z} \sin(\theta - \phi) \quad \text{where} \quad A = \frac{V_m}{Z} \sin(\alpha) \quad \phi = \tan^{-1} \frac{wL}{R}$$

Total solution :

where

$$i(\theta) = \frac{V_m}{Z} \sin(\phi) e^{\frac{R\theta}{wL}} + \frac{V_m}{Z} \sin(\theta - \phi)$$

FINALLY :

$$V_{dc} = \frac{1}{2\pi} \int_0^\delta V_m \sin(wt) dt$$

3- Single Phase half-wave rectifier (R-L Load)(with free wheeling diode)

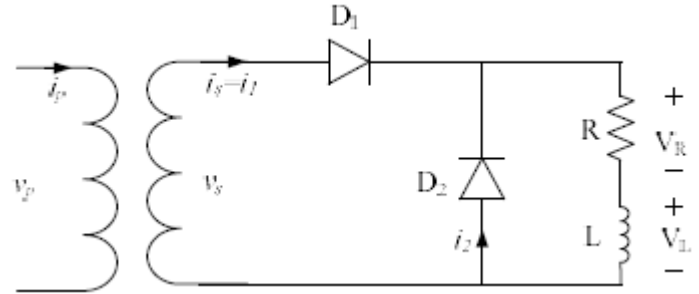
For $0 < \theta < \pi$

$$i_1(\theta) = Ae^{\frac{R\theta}{\omega L}} + \frac{V_m}{Z} \sin(\theta - \phi)$$

To calculate A, the value of $i_1(0) = i_2(2\pi) = I_2$

$$I_2 = Ae^{-0} + \frac{V_m}{Z} \sin(-\phi)$$

$$\therefore A = I_2 + \frac{V_m}{Z} \sin \phi$$



$$i_1(\theta) = \left(\frac{V_m}{Z} \sin \phi + I_2 \right) e^{\frac{R\theta}{\omega L}} + \frac{V_m}{Z} \sin(\theta - \phi)$$

The value of $i_1(\pi) = i_2(\pi) = I_1$

$$I_1 = \left(\frac{V_m}{Z} \sin \phi + I_2 \right) e^{\frac{R\pi}{\omega L}} + \frac{V_m}{Z} \sin \phi$$

For $\pi < \theta < 2\pi$

$$i_2(\theta) = I_1 e^{\frac{R}{\omega L}(\theta - \pi)}$$

The value of $i_2(2\pi) = i_1(0) = I_2$

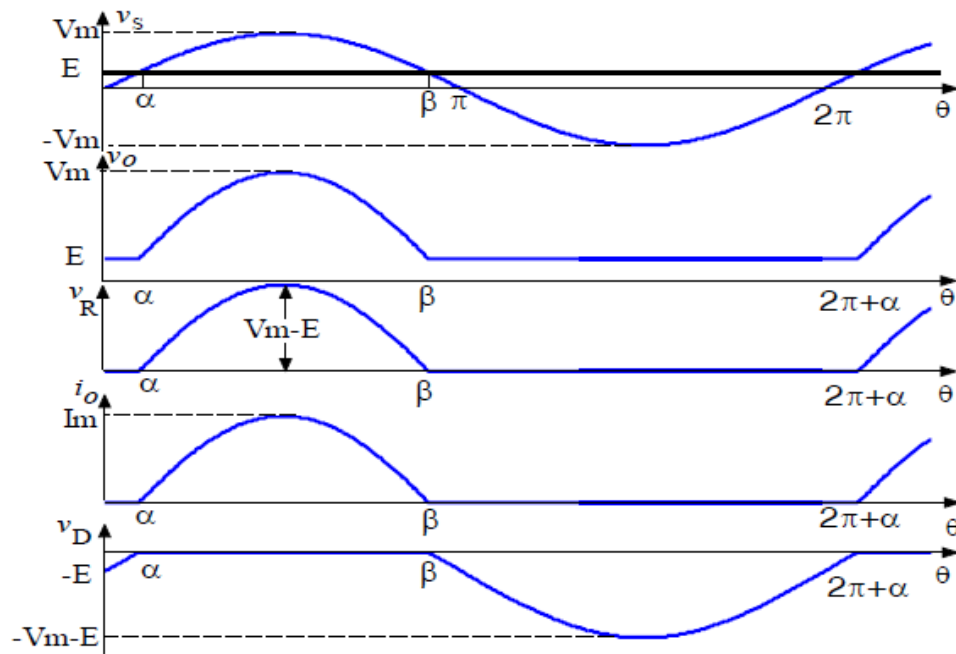
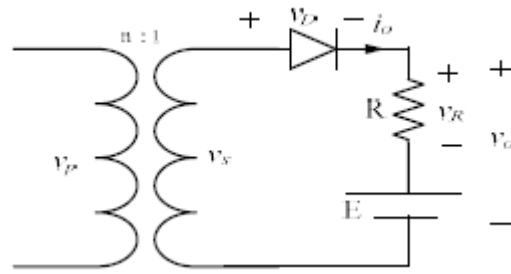
$$I_2 = I_1 e^{\frac{R}{\omega L}(2\pi - \pi)} = I_1 e^{\frac{R\pi}{\omega L}}$$

$$I_2 = \left\{ \left(\frac{V_m}{Z} \sin \phi + I_2 \right) e^{\frac{R\theta}{\omega L}} + \frac{V_m}{Z} \sin(\pi - \phi) \right\} e^{\frac{R\pi}{\omega L}}$$

$$I_2 = \frac{\frac{V_m}{Z} \sin \phi e^{\frac{R\pi}{\omega L}} (1 + e^{\frac{R\pi}{\omega L}})}{(1 - e^{\frac{R\pi}{\omega L}})}$$

$$\therefore I_1 = \left(\frac{V_m}{Z} \sin \phi + I_2 \right) e^{\frac{R\pi}{\omega L}} + \frac{V_m}{Z} \sin \phi$$

4- Single Phase half-wave rectifier (with Battery Charger)



$$v_R = V_m \sin(\theta) - E \quad \implies \quad i_o = \frac{v_R}{R} = \frac{V_m \sin(\theta) - E}{R}$$

$$\begin{aligned} I_{dc} &= \frac{1}{T} \int_{\alpha}^{\beta} i_o d\theta = \frac{1}{2\pi} \int_{\alpha}^{\pi-\alpha} \frac{V_m \sin(\theta) - E}{R} d\theta = \frac{1}{2\pi R} \left[\int_{\alpha}^{\pi-\alpha} V_m \sin(\theta) d\theta - \int_{\alpha}^{\pi-\alpha} E d\theta \right] \\ &= \frac{1}{2\pi R} \{ V_m [-\cos(\pi - \alpha) + \cos \alpha] - E[\pi - \alpha - \alpha] \} = \frac{2V_m \cos \alpha + 2\alpha E - \pi E}{2\pi R} \end{aligned}$$

$$P_{dc} = EI_{dc} = \frac{2EV_m \cos \alpha + 2\alpha E^2 - \pi E^2}{2\pi R} \quad \text{battery charging power}$$

$$t = \frac{W}{P_{dc}} \quad \text{battery charging time}$$

$$P_R = I_{rms}^2 2R$$

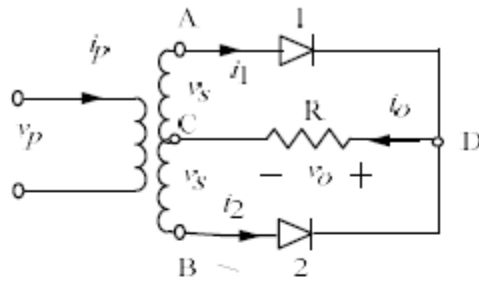
$$\% \eta = \frac{P_{dc}}{P_{dc} + P_R} \times 100$$

then : circuit efficiency :

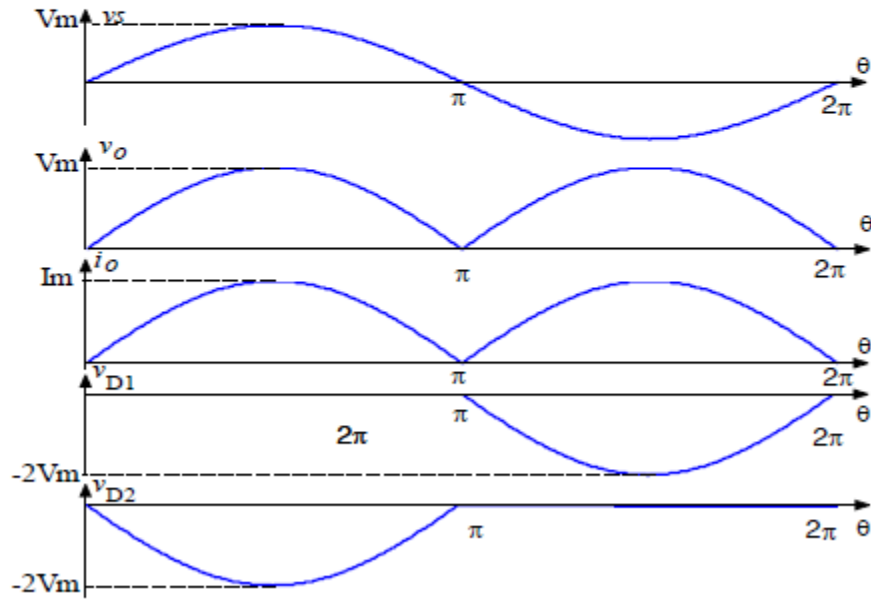
$$\begin{aligned} I_{rms} &= \sqrt{\frac{1}{T} \int_{\alpha}^{\beta} i_o^2 d\theta} = \sqrt{\frac{1}{2\pi R^2} \left[\int_{\alpha}^{\pi-\alpha} V_m^2 \sin^2 \theta d\theta - \int_{\alpha}^{\pi-\alpha} 2V_m E \sin \theta d\theta + \int_{\alpha}^{\pi-\alpha} E^2 d\theta \right]} \\ &= \sqrt{\frac{1}{2\pi R^2} \left\{ \frac{V_m^2}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) \Big|_{\alpha}^{\pi-\alpha} + 2V_m E (\cos \theta) \Big|_{\alpha}^{\pi-\alpha} + E^2 (\theta) \Big|_{\alpha}^{\pi-\alpha} \right\}} \\ &= \sqrt{\frac{1}{2\pi R^2} \left\{ \left(\frac{V_m^2}{2} + E^2 \right) (\pi - 2\alpha) + \frac{V_m^2}{2} \sin 2\alpha - 4V_m E \cos \alpha \right\}} \end{aligned}$$

$$PIV = E + V_m$$

5- Center-Tapped AC Rectifier



(a)



(b)

For $0 < \theta < \pi$

$$i_o = \frac{v_s}{R} = \frac{V_m}{R} \sin \theta, \quad i_1 = i_o, \quad i_2 = 0$$

For $\pi < \theta < 2\pi$

$$i_o = \frac{-v_s}{R} = -\frac{V_m}{R} \sin \theta, \quad i_1 = 0, \quad i_2 = i_o$$

$$V_{dc} = \frac{1}{T} \int_0^T v_o \, d\theta = \frac{1}{\pi} \int_0^\pi V_m \sin \theta \, d\theta = \frac{V_m}{\pi} [-\cos \theta]_0^\pi = \frac{V_m}{\pi} [-\cos \pi + \cos 0] = \frac{2V_m}{\pi}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

$$P_{dc} = I_{dc} V_{dc} = \frac{4V_m^2}{\pi^2 R}$$

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{T} \int_0^T v_o^2 \, d\theta} = \sqrt{\frac{1}{\pi} \int_0^\pi V_m^2 \sin^2 \theta \, d\theta} = \sqrt{\frac{V_m^2}{\pi} \int_0^\pi \frac{1}{2} (1 - \cos 2\theta) \, d\theta} \\ &= \sqrt{\frac{V_m^2}{2\pi} [\theta - \frac{1}{2} \sin 2\theta]_0^\pi} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} [\pi - 0 - \frac{1}{2} \sin 2\pi + \frac{1}{2} \sin 2\pi]} \\ &= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} [\pi - 0]} = \frac{V_m}{\sqrt{2}} \end{aligned}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2}R}$$

$$I_R = \sqrt{\frac{1}{T} \int_0^T i_1^2 \, d\theta} = \sqrt{\frac{1}{2\pi} \int_0^\pi \frac{V_m^2}{R^2} \sin^2 \theta \, d\theta} = \frac{V_m}{2R} = \frac{I_{rms}}{\sqrt{2}}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m^2}{2R}$$

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\left(\frac{4V_m^2}{\pi^2 R}\right)}{\left(\frac{V_m^2}{2R}\right)} \times 100 = \frac{8}{\pi^2} \times 100 = 81\%$$

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} = \sqrt{\frac{\left[\frac{V_m^2}{2}\right]}{\left[\frac{4V_m^2}{\pi^2}\right]} - 1} = \sqrt{\frac{\pi^2}{8} - 1} = 0.4834$$

$$S_T = S_1 + S_2 = \frac{V_m^2}{\sqrt{2}R}$$

$$TUF = \frac{P_{dc}}{S} = \frac{\left[\frac{4V_m^2}{\pi^2 R}\right]}{\left[\frac{V_m^2}{\sqrt{2}R}\right]} = \frac{4\sqrt{2}}{\pi^2} = 0.5732$$

The advantages of center-tapped rectifier are:

1. The rectifier efficiency is 81 %, i.e., twice that of half-wave rectifier.
2. The ripple factor is lower than that of half-wave rectifier.
3. The transformer utilization factor is greater than that of half-wave rectifier.
4. The problem of DC saturation in transformer secondary side is minimum, because the current flows in the opposite direction with respect to the center point of the transformer secondary.

The disadvantages of center-tapped rectifier are:

1. An additional center-tapped winding is required in the transformer.
2. The PIV is twice that of half-wave rectifier, so that it is required diodes with breakdown voltage twice that of diode that used in half-wave rectifier.
3. The transformer utilization factor is low because only one-half of the secondary winding works at a time.

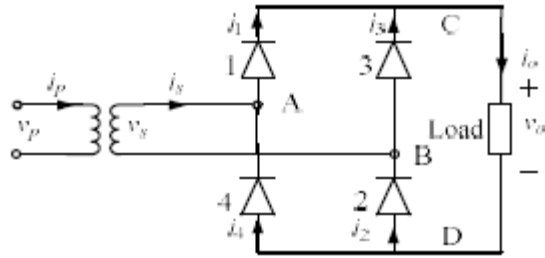
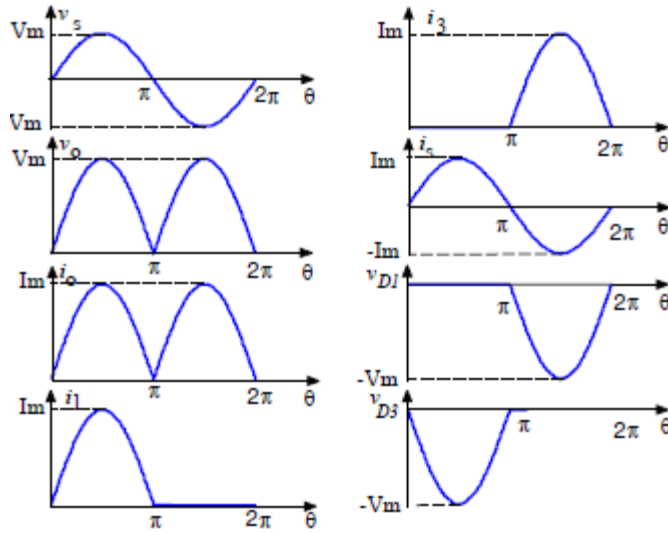


Fig. (3.12.a) Electric circuit



For $0 < \theta < \pi$

$$i_o = \frac{v_s}{R} = \frac{V_m}{R} \sin \theta$$

$$i_1 = i_2 = i_o$$

$$i_3 = i_4 = 0$$

For $\pi < \theta < 2\pi$

$$i_o = \frac{-v_s}{R} = \frac{V_m}{R} \sin \theta$$

$$i_1 = i_2 = 0$$

$$i_3 = i_4 = i_o$$

$$V_{dc} = \frac{1}{T} \int_0^T v_o d\theta = \frac{1}{\pi} \int_0^\pi V_m \sin \theta d\theta = \frac{V_m}{\pi} [-\cos \theta]_0^\pi = \frac{V_m}{\pi} [-\cos \pi + \cos 0] = \frac{2V_m}{\pi}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

$$I_A = \frac{1}{T} \int_0^T i_1 d\theta = \frac{1}{2\pi} \int_0^\pi \frac{V_m}{R} \sin \theta d\theta = \frac{V_m}{\pi R} = \frac{I_{dc}}{2}$$

$$P_{dc} = I_{dc} V_{dc} = \frac{4V_m^2}{\pi^2 R}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v_o^2 d\theta} = \sqrt{\frac{1}{\pi} \int_0^\pi V_m^2 \sin^2 \theta d\theta} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2}R}$$

$$I_R = \sqrt{\frac{1}{T} \int_0^T i_1^2 d\theta} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \frac{V_m^2}{R^2} \sin^2 \theta d\theta} = \frac{V_m}{2R} = \frac{I_{rms}}{\sqrt{2}}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m^2}{2R}$$

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\left(\frac{4V_m^2}{\pi^2 R}\right)}{\left(\frac{V_m^2}{2R}\right)} \times 100 = \frac{8}{\pi^2} \times 100 = 81\%$$

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} = \sqrt{\frac{\left[\frac{V_m^2}{2}\right]}{\left[\frac{4V_m^2}{\pi^2}\right]} - 1} = \sqrt{\frac{\pi^2}{8} - 1} = 0.4834$$

$$v_{D_3} = v_{D_4} = -v_s = -V_m \sin \theta$$

$$PIV = \max |v_{D_3}| = V_m$$

$$S = \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}R} = \frac{V_m^2}{2R} \quad TUF = \frac{P_{dc}}{S} = \frac{\left[\frac{4V_m^2}{\pi^2 R}\right]}{\left[\frac{V_m^2}{2R}\right]} = \frac{8}{\pi^2} = 0.8106$$

7- Single-phase full wave bridge rectifier R-L load

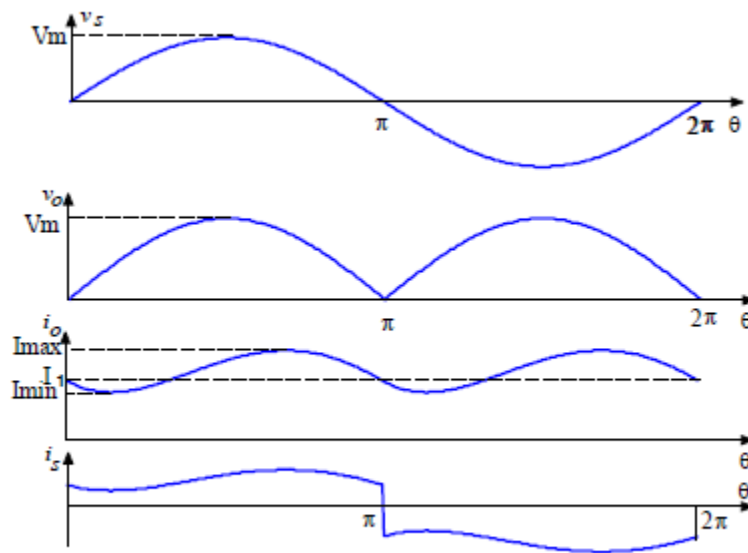
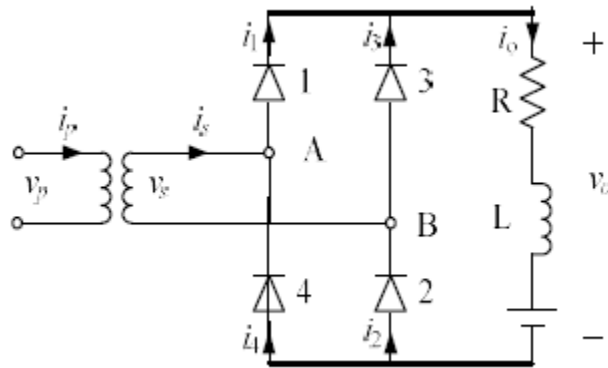


Fig. (3.16) Full-wave rectifier with R-L Load



$$i(\omega t) = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{E}{R} + A e^{-\frac{R}{L}t}$$

Where, $Z = \sqrt{R^2 + (\omega L)^2}$,

$$\phi = \tan^{-1} \frac{\omega L}{R},$$

$$A = \text{constant}$$

$$i(\pi) = I_1$$

$$I_1 = \frac{V_m}{Z} \sin(\pi - \phi) + A e^{-\frac{R}{L} \frac{\pi}{\omega}}$$

$$\therefore A = \left(I_1 + \frac{E}{R} - \frac{V_m}{Z} \sin \phi \right) e^{\frac{R}{L} \frac{\pi}{\omega}}$$

$$i(\omega t) = \frac{V_m}{Z} \sin(\omega t - \phi) + \left[\left(I_1 + \frac{E}{R} - \frac{V_m}{Z} \sin \phi \right) e^{\frac{R \pi}{L \omega}} \right] e^{-\frac{R}{L} t} - \frac{E}{R}$$

$$i(0) = i(\pi) = I_1$$

$$i(0) = I_1 = \frac{V_m}{Z} \sin(0 - \phi) + \left[\left(I_1 + \frac{E}{R} - \frac{V_m}{Z} \sin \phi \right) e^{\frac{R \pi}{L \omega}} \right] e^{-\frac{R}{L} 0} - \frac{E}{R}$$

$$\therefore I_1 \left(1 - e^{\frac{R \pi}{L \omega}} \right) = -\frac{V_m}{Z} \sin(\phi) + \left(\frac{E}{R} - \frac{V_m}{Z} \sin \phi \right) e^{\frac{R \pi}{L \omega}} - \frac{E}{R}$$

$$\therefore I_1 = \frac{\left(-\frac{V_m}{Z} \sin(\phi) + \left(\frac{E}{R} - \frac{V_m}{Z} \sin \phi \right) e^{\frac{R \pi}{L \omega}} - \frac{E}{R} \right)}{\left(1 - e^{\frac{R \pi}{L \omega}} \right)}$$

8- Three-phase full-wave rectifier

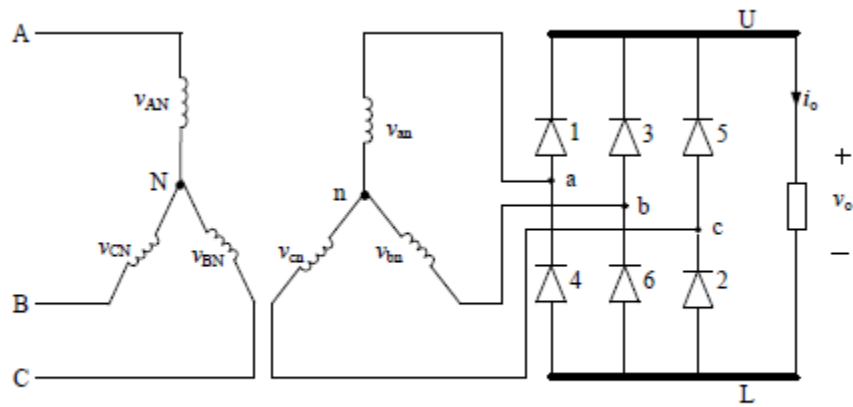
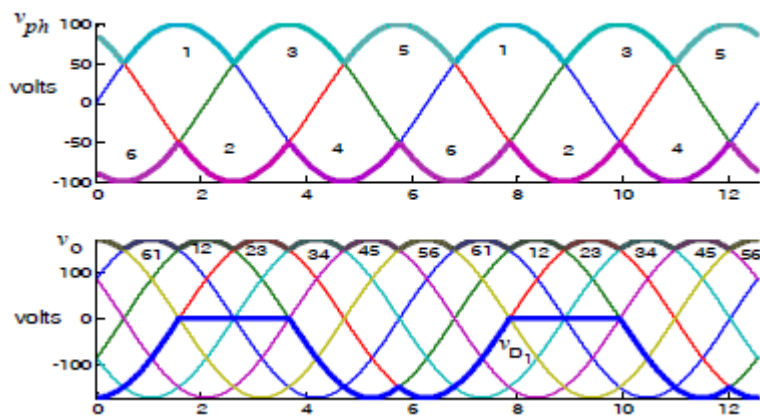
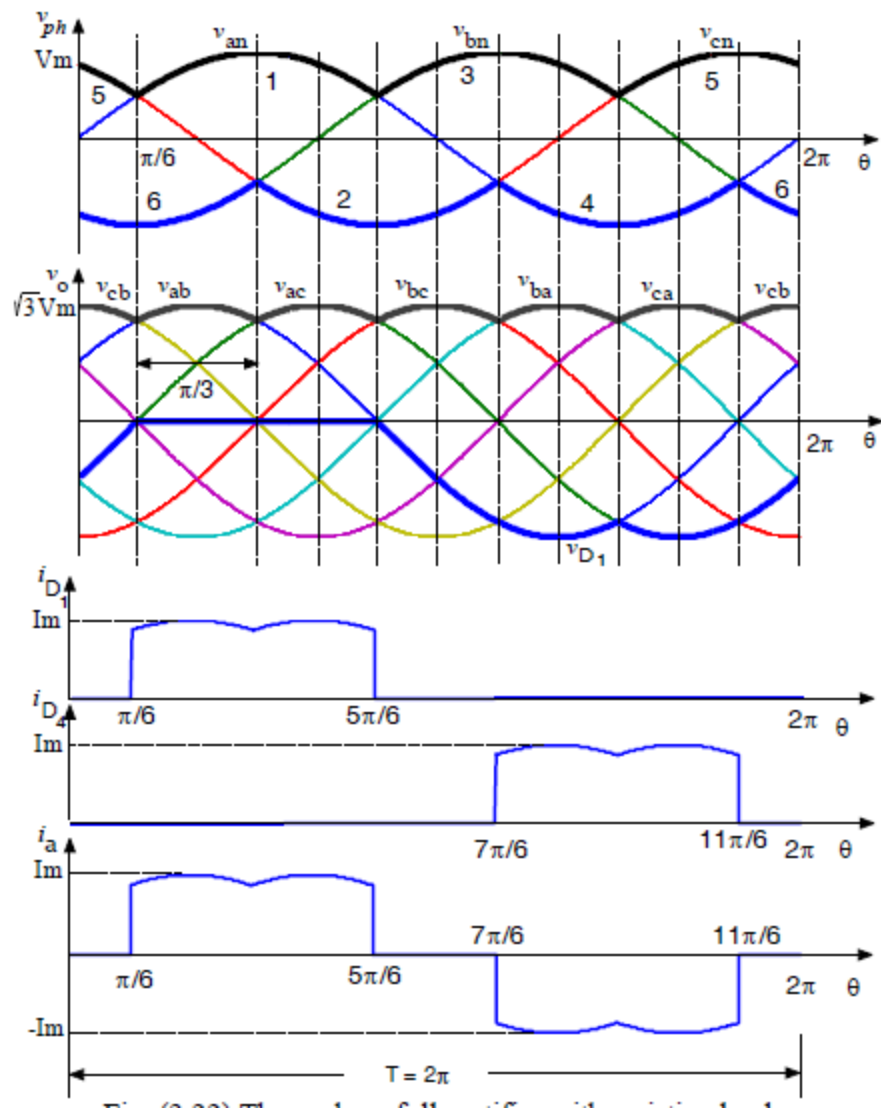


Fig. (3.20) Three-phase bridge rectifier circuit.



Phase voltage	Line voltage
$v_{an} = V_m \sin \theta$	$v_{ab} = \sqrt{3}V_m \sin(\theta + 30^\circ)$
$v_{bn} = V_m \sin(\theta - 120^\circ)$	$v_{bc} = \sqrt{3}V_m \sin(\theta - 90^\circ)$
$v_{cn} = V_m \sin(\theta + 120^\circ)$	$v_{ca} = \sqrt{3}V_m \sin(\theta - 210^\circ)$

$$\begin{aligned}
 V_{dc} &= \frac{3}{\pi} \int_{\pi/6}^{\pi/2} v_{ab} d\theta = \frac{3}{\pi} \int_{\pi/6}^{\pi/2} \sqrt{3}V_m \sin(\theta + 30^\circ) d\theta \\
 &= \frac{3\sqrt{3}V_m}{\pi} [-\cos(\theta + 30^\circ)]_{\pi/6}^{\pi/2} = \frac{3\sqrt{3}V_m}{\pi} [-\cos 120^\circ + \cos 60^\circ]_{\pi/6}^{\pi/2} \\
 &= \frac{3\sqrt{3}V_m}{\pi} = 1.654V_m
 \end{aligned}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R} = 1.654 \frac{V_m}{R}$$

$$I_A = \frac{I_{dc}}{3} = \frac{\sqrt{3}V_m}{\pi R} = 0.5513 \frac{V_m}{R}$$

$$\begin{aligned}
 V_{rms} &= \sqrt{\frac{3}{\pi} \int_{\pi/6}^{\pi/2} v_{ab}^2 d\theta} = \sqrt{\frac{3}{\pi} \int_{\pi/6}^{\pi/2} 3V_m^2 \sin^2(\theta + 30^\circ) d\theta} \\
 &= \sqrt{\frac{9V_m^2}{2\pi} \left[\theta - \frac{1}{2} \sin(2\theta + 60^\circ) \right]_{\pi/6}^{\pi/2}} = \sqrt{\frac{9V_m^2}{2\pi} \left[\frac{\pi}{3} - \frac{1}{2} (\sin 240^\circ - \sin 120^\circ) \right]} \\
 &= \sqrt{\frac{9V_m^2}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \right]} = 1.6554V_m
 \end{aligned}$$

$$I_{rms} = \frac{V_{rms}}{R} = 1.6554 \frac{V_m}{R} \quad \text{////} \quad I_m = \frac{\sqrt{3}V_m}{R}$$

$$I_s = \sqrt{2}I_R = 1.3516 \frac{V_m}{R} = 0.7804I_m$$

$$P_{dc} = V_{dc}I_{dc} = \frac{27V_m^2}{\pi^2 R} = 2.73567 \frac{V_m^2}{R}$$

$$\% \eta = \frac{P_{ac}}{P_{dc}} \times 100 = 99.83\%$$

$$FF = \frac{V_{rms}}{V_{dc}} = 1.00085$$

FF is almost equal to unity as:

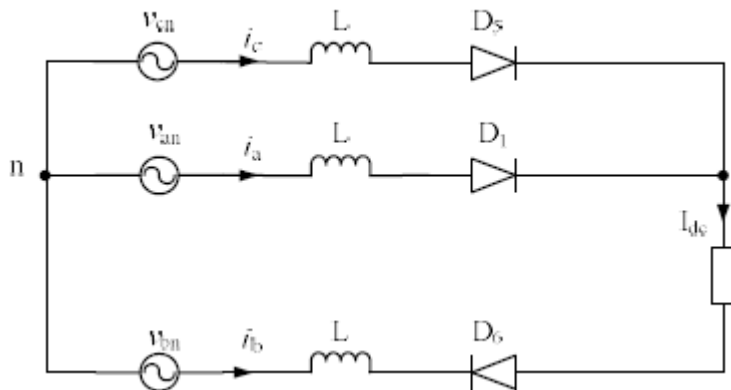
$$RF = \sqrt{FF^2 - 1} = 0.0411 = 4.11\%$$

1.2.3.3. THRESHOLD VALUE IS:

$$S = 3V_s I_s = 3 \times \frac{V_m}{\sqrt{2}} \times 1.3516 \frac{V_m}{R} = 2.86718 \frac{V_m^2}{R}$$

$$TUF = \frac{2.73567 \frac{V_m^2}{R}}{2.86718 \frac{V_m^2}{R}} = 0.954 = 95.4\%$$

9- Three-phase full-wave rectifier with commutation



Applying KCL at node x yields:

$$i_a + i_c = I_{dc}$$

$$\therefore \frac{di_a}{dt} = -\frac{di_c}{dt}$$

Applying KVL for upper closed loop yields:

$$v_{an} - L \frac{di_a}{dt} + L \frac{di_c}{dt} - v_{cn} = 0$$

Substitute from (3.12) into (3.13) yields:

$$L \frac{di_a}{dt} = \frac{v_{an} - v_{cn}}{2}$$

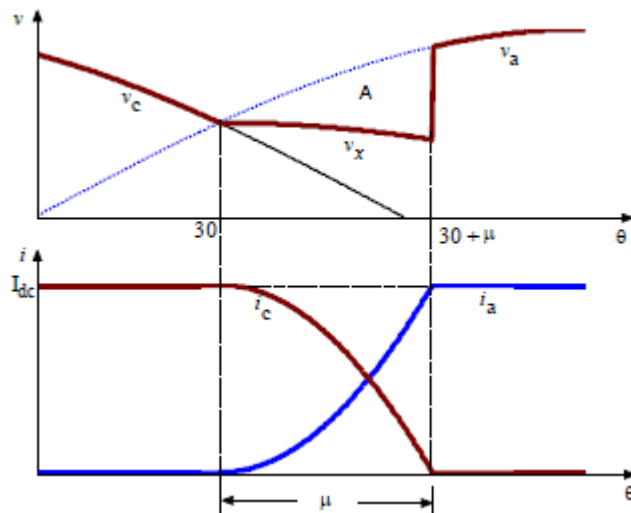
The potential at node x is:

$$v_x = v_{an} - L \frac{di_a}{dt}$$

Substitute from (3.14) into (3.15) yields:

$$v_x = \frac{v_{an} + v_{cn}}{2}$$

Reduction Area



$$\begin{aligned}
 A &= \int_{30}^{30+\mu} (v_{an} - v_x) d\omega t = \int_{30}^{30+\mu} \frac{v_{an} - v_{cn}}{2} d\omega t \\
 &= \int_0^{I_{dc}} \omega L \frac{di}{dt} dt = \int_0^{I_{dc}} \omega L di = \omega L I_{dc}
 \end{aligned}$$

The reduction of output voltage will be repeated μ times in one cycle. The average value of reduction voltage is:

$$\Delta V_{dc} = \frac{\omega L I_{dc}}{\pi/3} = \frac{3 L I_{dc} (2\pi f)}{\pi} = 6 f L I_{dc}$$

Then the average value of output voltage is:

$$V_{dc} = V_{dco} - \Delta V_{dc} = \frac{3\sqrt{3}V_m}{\pi} - 6 f L I_{dc}$$

Phase Currents During Commutation and calculating μ

$$i_b = -I_{dc}$$

$$\therefore \frac{di_a}{dt} = \frac{v_{an} - v_{cn}}{2L}$$

$$\therefore \int_0^{i_a} di_a = \int_{30}^{\omega t} \frac{v_{an} - v_{cn}}{2\omega L} d\omega t$$

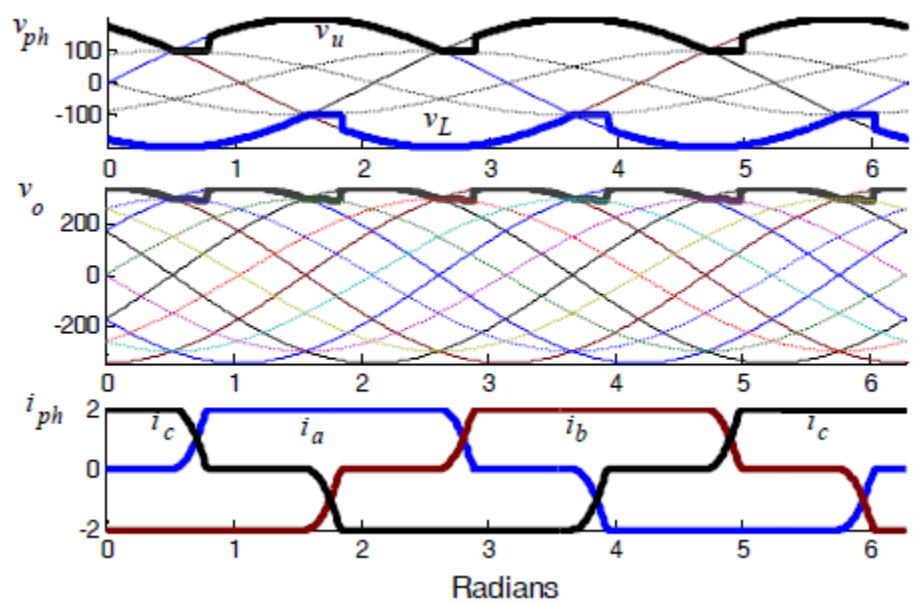
$$i_a = \int_{30}^{\omega t} \frac{\sqrt{3}V_m \sin(\omega t - 30)}{2\omega L} d\omega t = \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 30)]$$

$$i_c = I_{dc} - i_a = I_{dc} - \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 30)]$$

To determine value of commutation angle μ , substitute about $\omega t = 30 + \mu$ in (3.21) and $i_a = I_{dc}$ yields:

$$I_{dc} = \frac{\sqrt{3}V_m}{2\omega L} (1 - \cos \mu)$$

$$\therefore \mu = \cos^{-1} \left(1 - \frac{2\omega L I_{dc}}{\sqrt{3}V_m} \right)$$



SUMMARY:

	$1 - \phi$ halfwave	$1 - \phi$ halfwave w battery charger	Center tapped"
V_{dc}	$\frac{V_m}{\pi}$		$= \frac{2V_m}{\pi}$
I_{dc}	$\frac{V_m}{\pi R}$	$\frac{2V_m \cos \alpha + 2\alpha E - \pi E}{2\pi R}$	$\frac{2V_m}{\pi R}$
P_{dc}	$\frac{V_m^2}{\pi^2 R}$	$\frac{2EV_m \cos \alpha + 2\alpha E^2 - \pi E^2}{2\pi R}$	$\frac{4V_m^2}{\pi^2 R}$
V_{rms}	$\frac{V_m}{2}$		$= \frac{V_m}{\sqrt{2}}$
I_{rms}	$\frac{V_m}{2R}$	$\sqrt{\frac{1}{2\pi R^2} \left\{ \left(\frac{V_m^2}{2} + E^2 \right) (\pi - 2\alpha) + \frac{V_m^2}{2} \sin 2\alpha - 4V_m E \cos \alpha \right\}}$	$= \frac{V_m}{\sqrt{2}R}$
I_R			$\frac{I_{rms}}{\sqrt{2}}$
P_{rms}	$\frac{V_m^2}{4R}$	$I_{rms}^2 2R$	$= \frac{V_m^2}{2R}$
$\% \eta = P_{dc}/P_{ac}$	40.53%		81%
FF (form factor) $= \frac{V_{rms}}{V_{dc}}$	$\frac{\pi}{2}$		
$Ripple$ factor RF $= \sqrt{(FF)^2 - 1}$	1.2114		0.4834
$TUF = \frac{P_{dc}}{S}$	0.2867		0.5732
PIV	V_m	$E + V_m$	$2V_m$

In DC FF=1 RF = 0

	1 – ϕ full wave R	3 – ϕ Full wave		
V_{dc}	$\frac{2V_m}{\pi}$	$1.654V_m$		
I_{dc}	$\frac{2V_m}{\pi R}$	$1.654 \frac{V_m}{R}$		
I_A		$0.5513 \frac{V_m}{R}$		
P_{dc}	$\frac{4V_m^2}{\pi^2 R}$	$2.73567 \frac{V_m^2}{R}$		
V_{rms}	$\frac{V_m}{\sqrt{2}}$	$1.6554V_m$		
I_{rms}	$\frac{V_m}{\sqrt{2}R}$	$1.6554 \frac{V_m}{R}$		
I_s		$0.7804I_m$		
I_R	$\frac{I_{rms}}{\sqrt{2}}$	$0.5518I_m$		
P_{rms}	$\frac{V_m^2}{2R}$	$2.74035 \frac{V_m^2}{R}$		
$\% \eta = P_{dc}/P_{ac}$	81%	99.83%		
FF (form factor) $= \frac{V_{rms}}{V_{dc}}$		1.00085		
Ripple factor RF $= \sqrt{(FF)^2 - 1}$	0.4834	0.0411		
$TUF = \frac{P_{dc}}{S}$	0.8106	0.954		
PIV	V_m	$\sqrt{3}V_m$		

Table (3.1) Phase currents during all intervals in three-phase full-wave bridge rectifier.

Interval		Phase currents		
From	To	i_a	i_b	i_c
0	$\pi/6$	0	$-I_{dc}$	I_{dc}
$\pi/6$	$\pi/6+\mu$	$\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 30^\circ)]$	$-I_{dc}$	$I_{dc} - \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 30^\circ)]$
$\pi/6+\mu$	$\pi/2$	I_{dc}	$-I_{dc}$	0
$\pi/2$	$\pi/2+\mu$	I_{dc}	$-I_{dc} + \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 90^\circ)]$	$-\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 90^\circ)]$
$\pi/2+\mu$	$5\pi/6$	I_{dc}	0	$-I_{dc}$
$5\pi/6$	$5\pi/6+\mu$	$I_{dc} - \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 150^\circ)]$	$\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 150^\circ)]$	$-I_{dc}$
$5\pi/6+\mu$	$7\pi/6$	0	I_{dc}	$-I_{dc}$
$7\pi/6$	$7\pi/6+\mu$	$-\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 210^\circ)]$	I_{dc}	$-I_{dc} + \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 210^\circ)]$
$7\pi/6+\mu$	$9\pi/6$	$-I_{dc}$	I_{dc}	0
$9\pi/6$	$9\pi/6+\mu$	$-I_{dc}$	$I_{dc} - \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 270^\circ)]$	$\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 270^\circ)]$
$9\pi/6+\mu$	$11\pi/6$	$-I_{dc}$	0	I_{dc}
$11\pi/6$	$11\pi/6+\mu$	$-I_{dc} + \frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 330^\circ)]$	$-\frac{\sqrt{3}V_m}{2\omega L} [1 - \cos(\omega t - 330^\circ)]$	I_{dc}
$11\pi/6+\mu$	2π	0	$-I_{dc}$	I_{dc}

Single phase half wave controlled (R load)

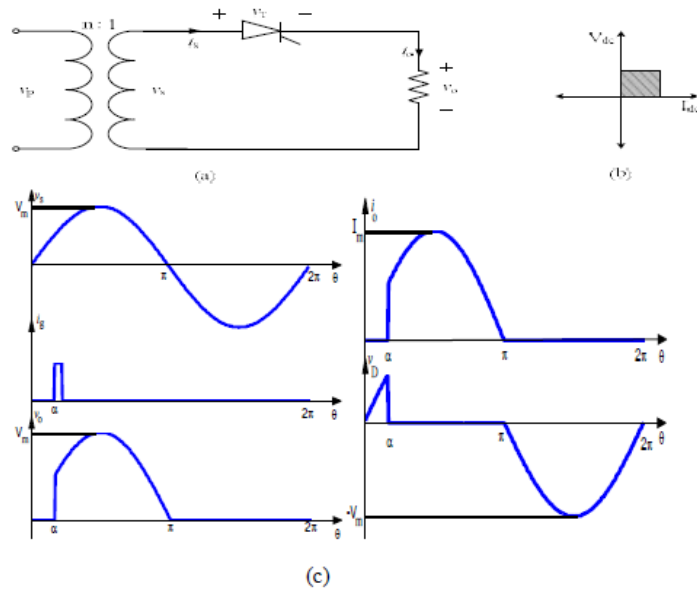


Fig. 4.1 Half-wave controlled rectifier with resistive load.

$$V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \theta d\theta = \frac{V_m}{2\pi} (-\cos \theta) \Big|_{\alpha}^{\pi} = \frac{V_m}{2\pi} (-\cos \pi + \cos \alpha) = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

$$\text{Min @ } \alpha = 180^\circ V_{dc} = 0 \quad ; \quad \text{Max @ } \alpha = 0 V_{dc} = \frac{V_m}{\pi}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R} (1 + \cos \alpha)$$

$$I_A = \frac{1}{T} \int_{\alpha}^{\pi} i_o d\theta = I_{dc} = \frac{V_m}{2\pi R} (1 + \cos \alpha)$$

$$P_{dc} = I_{dc} V_{dc} = \frac{V_m^2}{4\pi^2 R} (1 + \cos \alpha)^2$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \theta d\theta} = \sqrt{\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} \frac{1}{2} (1 - \cos 2\theta) d\theta} = \frac{V_m}{2} \sqrt{\frac{1}{\pi} [\pi - \alpha + \frac{1}{2} \sin 2\alpha]}$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R} \sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$$

$$I_R = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R^2} \sin^2 \theta d\theta} = I_{rms} = \frac{V_m}{2R} \sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m^2}{4\pi R^2} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)$$

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\frac{V_m^2}{4\pi^2 R} (1 + \cos \alpha)^2}{\frac{V_m^2}{4\pi R^2} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)} \times 100 = \frac{(1 + \cos \alpha)^2}{\pi (\pi - \alpha + \frac{1}{2} \sin 2\alpha)} \times 100$$

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}} \right]^2} - 1 = \sqrt{\frac{\frac{V_m^2}{4} \left(\frac{1}{\pi} [\pi - \alpha + \frac{1}{2} \sin 2\alpha] \right)}{\frac{V_m^2}{4\pi^2} (1 + \cos \alpha)^2}} - 1 = \sqrt{\frac{\pi (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}{(1 + \cos \alpha)^2}} - 1$$

$$PIV = \max |v_D| = V_m$$

$$S = V_s I_s = \frac{V_m}{\sqrt{2}} \times \frac{V_m}{2R} \sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)} = \frac{V_m^2}{2\sqrt{2}R} \sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$$

$$TUF = \frac{P_{dc}}{S} = \frac{\frac{V_m^2}{4\pi^2 R} (1 + \cos \alpha)^2}{\frac{V_m^2}{2\sqrt{2}R} \sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}} = \frac{1}{\sqrt{2}\pi^2} \frac{(1 + \cos \alpha)^2}{\sqrt{\frac{1}{\pi} (\pi - \alpha + \frac{1}{2} \sin 2\alpha)}}$$

$$v_T = v_s = V_m \sin \theta$$

Single Phase Half-Wave Controlled rectifier with R-L Load

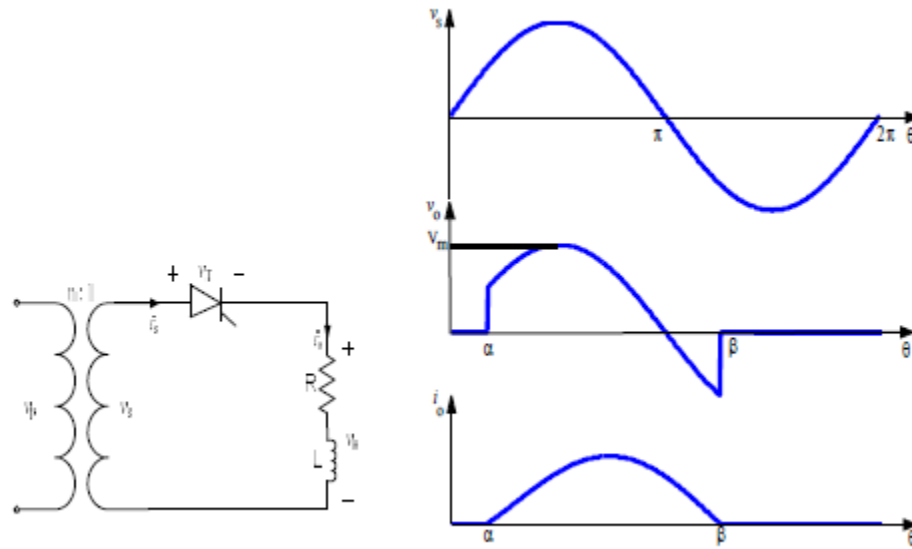


Fig. (4.3) half-wave controlled rectifier with R-L load

$$i(\theta) = i_t + i_s$$

$$i_s = \frac{V_m}{Z} \sin(\theta - \phi) \quad \text{and} \quad i_t = Ae^{\frac{R}{L}\theta}$$

$$i(\theta) = Ae^{\frac{R}{L}\theta} + \frac{V_m}{Z} \sin(\theta - \phi)$$

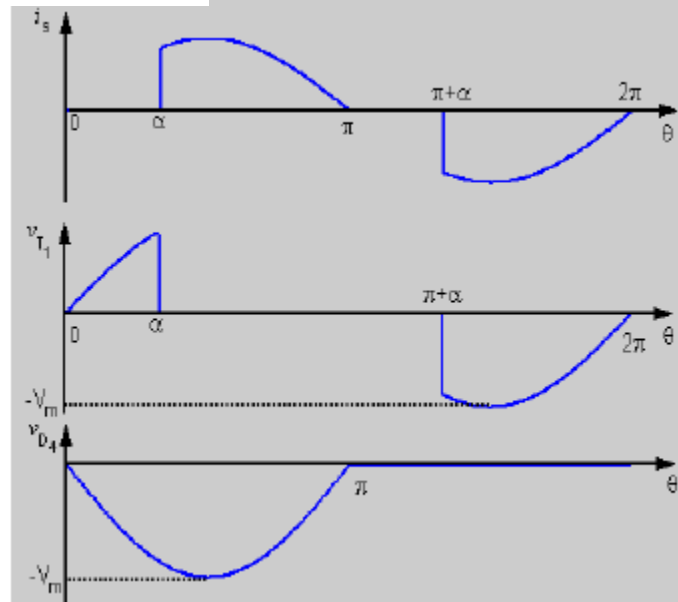
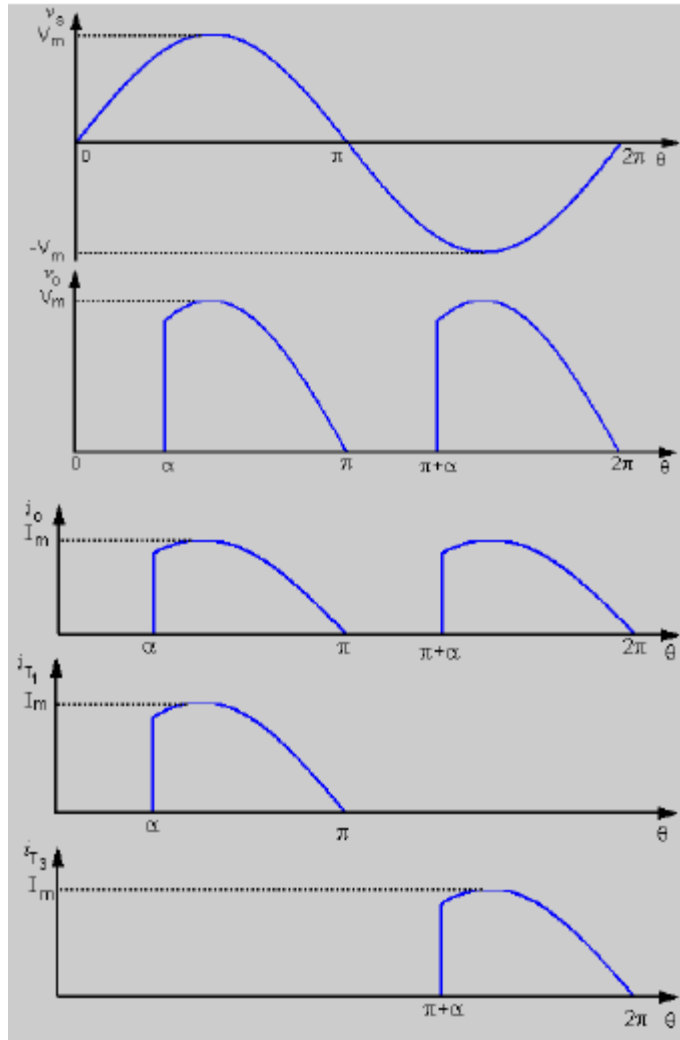
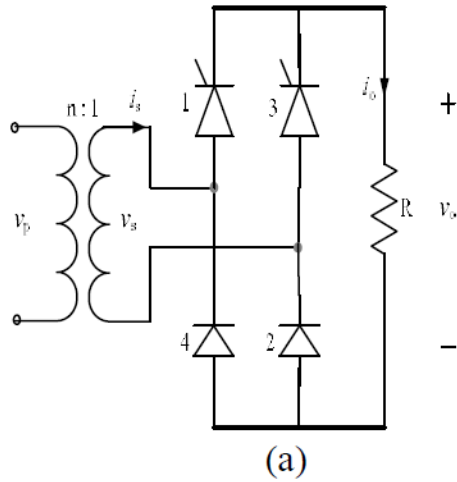
or

$$i(\theta) = \frac{V_m}{Z} \left\{ \sin(\theta - \phi) - \sin(\alpha - \phi) e^{\frac{R}{\omega L}(\alpha - \theta)} \right\} \quad \text{for } \alpha < \theta < \beta$$

To get value of β

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{R}{\omega L}(\alpha - \beta)}$$

Single-Phase Semiconverter with R load



$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R} (1 + \cos \alpha)$$

The average value of diode current can be calculated as follows:

$$I_A = \frac{1}{2\pi} \int_{\alpha}^{\pi} i_o d\theta = \frac{I_{dc}}{2} = \frac{V_m}{2\pi R} (1 + \cos \alpha)$$

Therefore the average output power is:

$$P_{dc} = I_{dc} V_{dc} = \frac{V_m^2}{\pi^2 R} (1 + \cos \alpha)^2$$

The rms voltage of out put voltage can be calculated as follows:

$$\begin{aligned} V_{rms} &= \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \theta d\theta} = \sqrt{\frac{V_m^2}{\pi} \int_{\alpha}^{\pi} \frac{1}{2} (1 - \cos 2\theta) d\theta} \\ &= \sqrt{\frac{V_m^2}{2\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_{\alpha}^{\pi}} = V_m \sqrt{\frac{1}{2\pi} \left(\pi - \alpha - \frac{1}{2} \sin 2\pi + \frac{1}{2} \sin 2\alpha \right)} \\ &= V_m \sqrt{\frac{1}{2\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]} \end{aligned}$$

Similarly, the rms value of output current is:

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{R} \sqrt{\frac{1}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)}$$

The rms value of diode current can be calculated as follows:

$$I_R = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m^2}{R^2} \sin^2 \theta d\theta} = \frac{I_{rms}}{\sqrt{2}} = \frac{V_m}{2R} \sqrt{\frac{1}{\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha \right]}$$

The output AC power is:

$$P_{ac} = V_{rms} I_{rms} = \frac{V_m^2}{2\pi R^2} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)$$

Therefore, the efficiency of full-wave center-tapped rectifier is:

$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\frac{V_m^2}{\pi^2 R} (1 + \cos \alpha)^2}{\frac{V_m^2}{2\pi R^2} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)} \times 100 = \frac{2(1 + \cos \alpha)^2}{\pi \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)} \times 100$$

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} = \sqrt{\frac{\frac{V_m^2}{2} \left(\frac{1}{\pi} \left[\pi - \alpha + \frac{1}{2} \sin 2\alpha\right]\right)}{\frac{V_m^2}{\pi^2} (1 + \cos \alpha)^2} - 1} = \sqrt{\frac{\pi(\pi - \alpha + \frac{1}{2} \sin 2\alpha)}{2(1 + \cos \alpha)^2} - 1}$$

$$f_o = 2f_{in}$$

The maximum inverse voltage across D_2 is:

$$PIV = \max |v_D| = V_m$$

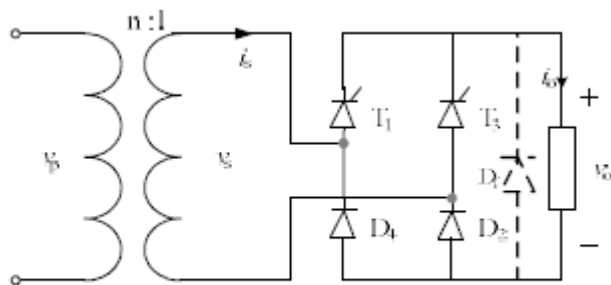
The VA of rectifier transformer is:

$$S = V_s I_s = \frac{V_m}{\sqrt{2}} \times \frac{V_m}{R} \sqrt{\frac{1}{2\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha\right)} = \frac{V_m^2}{\sqrt{2}R} \sqrt{\frac{1}{\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha\right)}$$

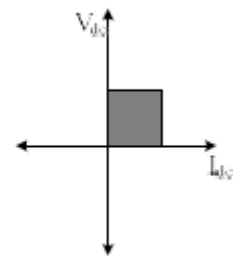
Therefore, the transformer utilization factor of half-wave rectifier is:

$$TUF = \frac{P_{dc}}{S} = \frac{\frac{V_m^2}{\pi^2 R} (1 + \cos \alpha)^2}{\frac{V_m^2}{\sqrt{2}R} \sqrt{\frac{1}{\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha\right)}} = \frac{\sqrt{2}}{\pi^2} \frac{(1 + \cos \alpha)^2}{\sqrt{\frac{1}{\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha\right)}}$$

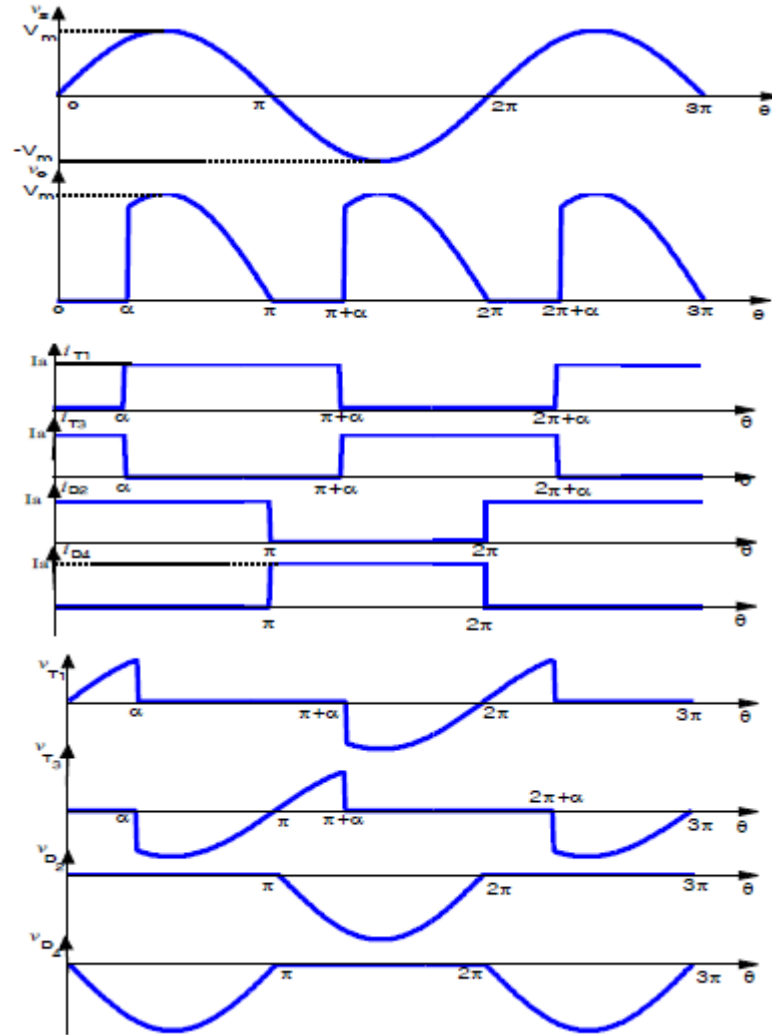
Single-Phase Semiconverter with R-L load without FD



(a)



(b)



To Plot V_{T1}, V_{T2}
 From eqn:-
 $V_s - V_{T1} + V_{T2} = 0$

To Plot V_{D2}, V_{D4}
 From eqn:-
 $V_s = V_{D2} - V_{D4}$

NOTE : firing angle of thyristor must equal to θ of R-L load . to avoid Ripp,e in output current

$$\therefore \alpha = \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

القوانين نفس القوانين پس ناخذ ف الاعتبار التيارات pure DC

$$V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

$$V_{rms} = V_m \sqrt{\frac{1}{2\pi} (\pi - \alpha + \frac{\sin 2\alpha}{2})}$$

$$I_{dc} = I_{rms} = I_a$$

$$I_A = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} I_a dt = \frac{I_a}{2\pi} \times \pi = \frac{I_a}{2}$$

$$I_R = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} I_a^2 dt} = \frac{I_a}{\sqrt{2}}$$

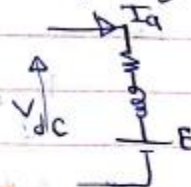
$$I_{s rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} I_a^2 dt} = I_a \sqrt{\frac{1}{\pi} (\pi - \alpha)} \quad \checkmark$$

$$= \sqrt{2} I_R \quad \times \times \quad (\text{الاختلاف فترة زمنية } T_1 \text{ من فترة تغذية})$$

→ To Find I_a :

$$V_{dc} = I_a R + L \frac{dI_a}{dt} + E$$

$$\therefore I_a = \frac{V_{dc} - E}{R}$$



$$P_{out} = I_a^2 R + E I_a = I_a V_{dc}$$

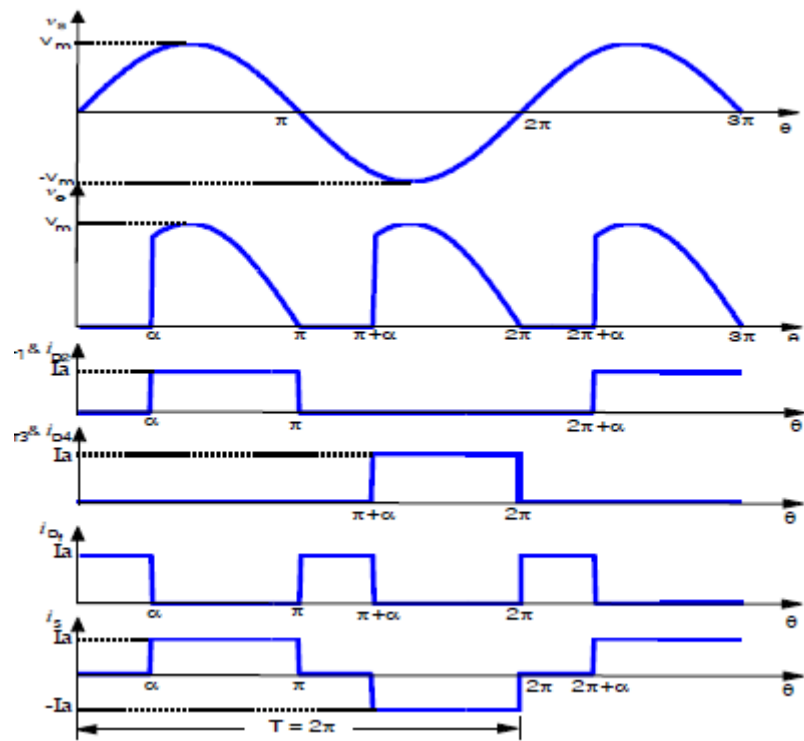
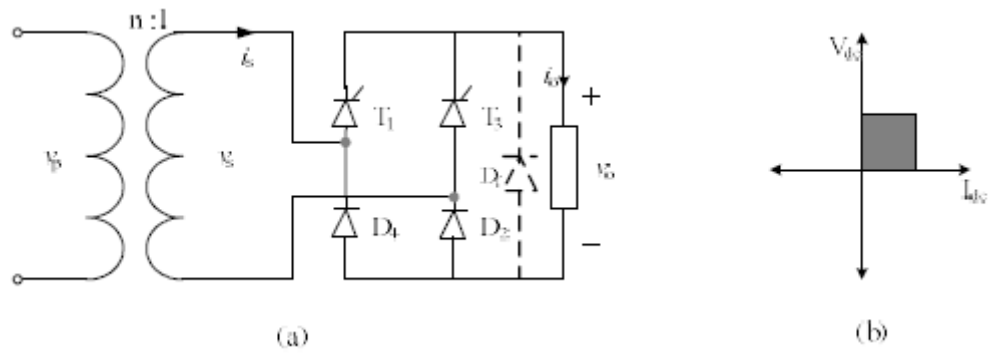
$$PF = \frac{P_{out}}{S_{in}} = \frac{V_{dc} I_a}{S_{in}}$$

$$T_{uF} = \frac{P_{dc}}{S_{in}} = \frac{V_{dc} I_a}{S_{in}}$$

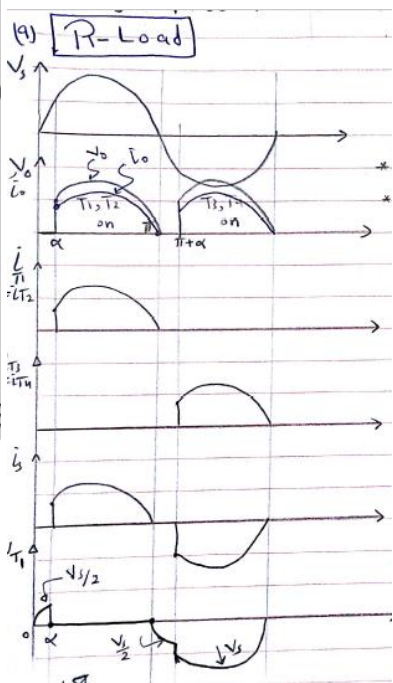
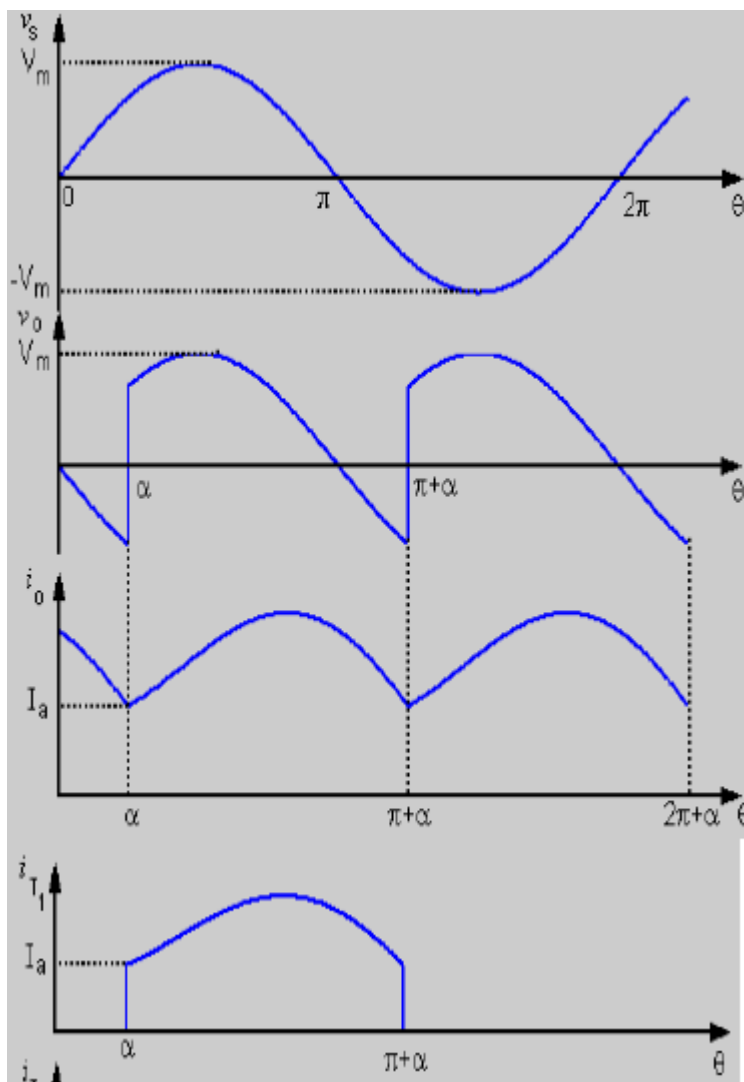
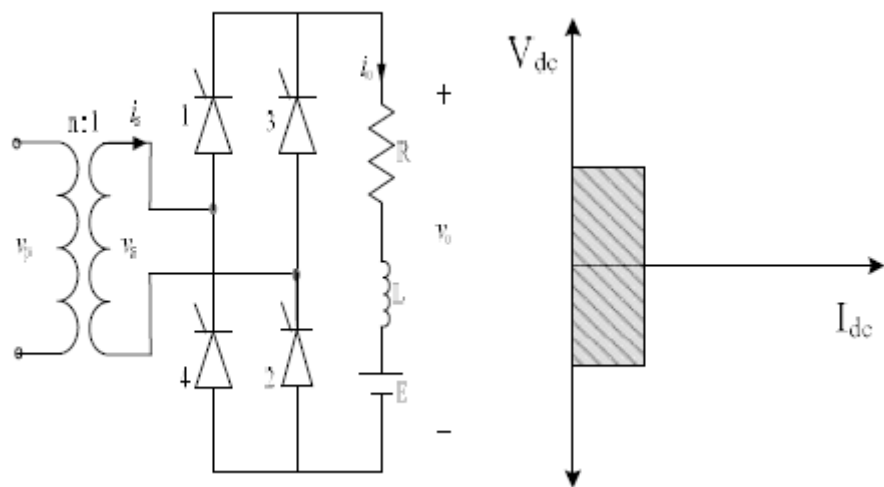
$$AT H.I.L \rightarrow T_{uF} = PF_{in}$$

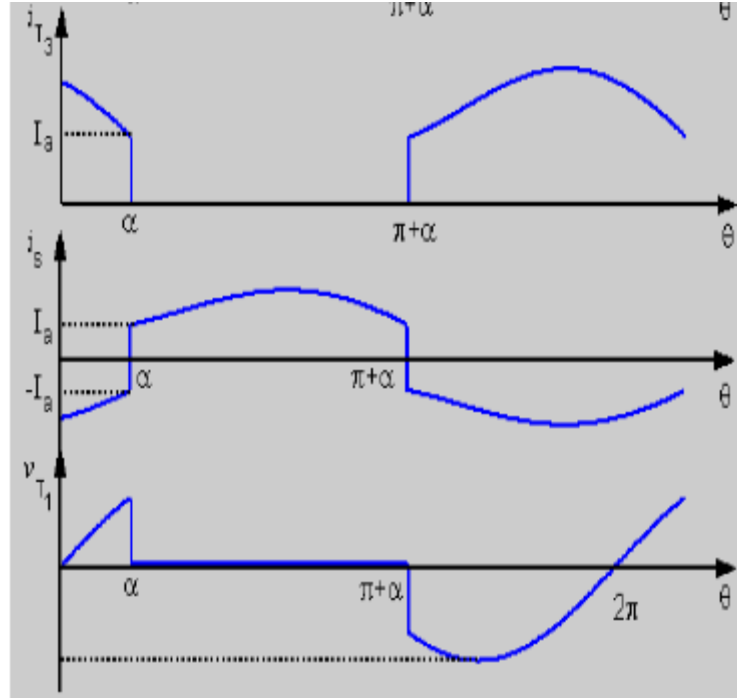
$$\text{because } P_{ac} = P_{dc}$$

Single-Phase Semiconverter with R-L load with FD



Single-Phase Full converter with R-L Load





mode 1 ($\theta \rightarrow \alpha : \pi + \alpha$)

$$\therefore i(\theta) = \frac{V_m}{Z} \sin(\theta - \phi) - \frac{E}{R} + A_1 e^{-\frac{R}{L}(\theta - \frac{\alpha}{\omega})}$$

$$\therefore A_1 = (I_a - \frac{V_m}{Z} \sin(\alpha - \phi) + \frac{E}{R})$$

$$i(\theta) = \frac{V_m}{Z} \sin(\theta - \phi) - \frac{E}{R} + \{I_a - \frac{V_m}{Z} \sin(\alpha - \phi) + \frac{E}{R}\} e^{-\frac{R}{L}(\theta - \frac{\alpha}{\omega})}$$

$$\therefore I_a = \frac{-\frac{V_m}{Z} \sin(\alpha - \phi) - \frac{E}{R} + \{-\frac{V_m}{Z} \sin(\alpha - \phi) + \frac{E}{R}\} e^{-\frac{R}{L}(\frac{\pi}{\omega})}}{(1 - e^{-\frac{R}{L}(\frac{\pi}{\omega})})}$$

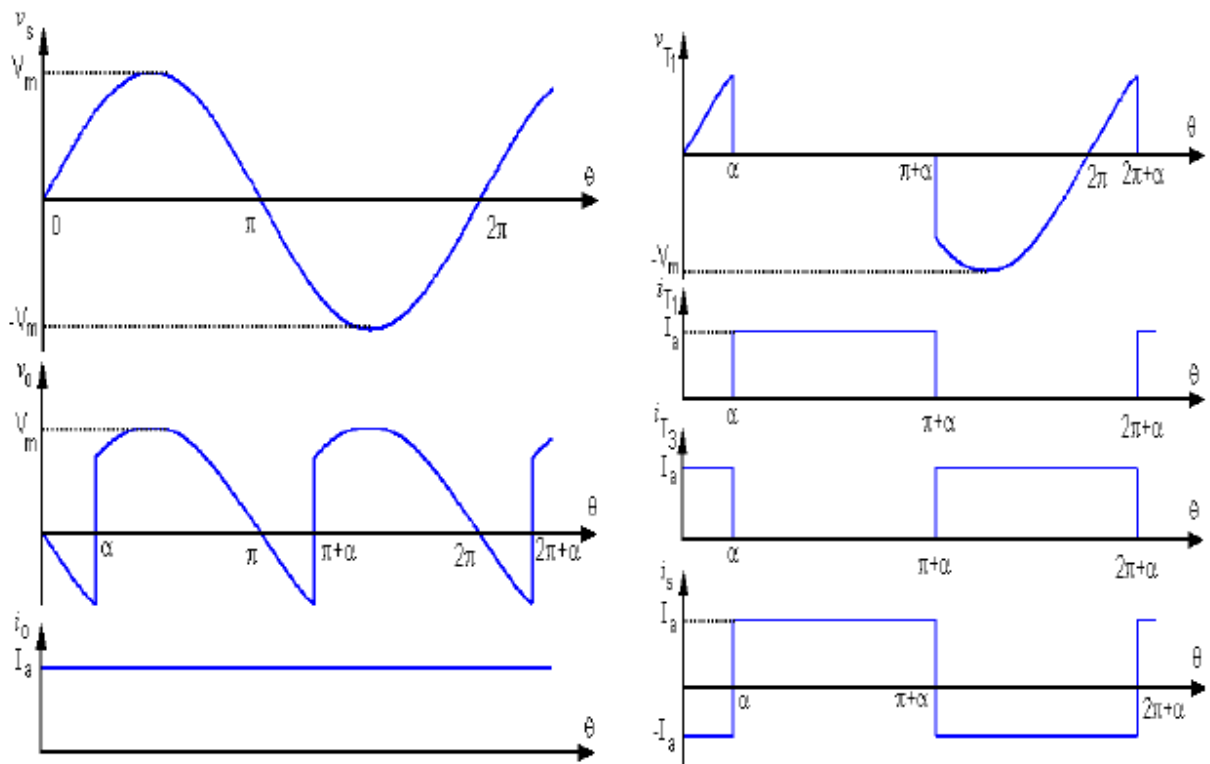
mode 2 ($\theta \rightarrow \pi + \alpha : 2\pi + \alpha$)

$$\therefore i(\theta) = -\frac{V_m}{Z} \sin(\theta - \phi) - \frac{E}{R} - A_2 e^{-\frac{R}{L}(\theta - \frac{\pi + \alpha}{\omega})}$$

$$A_2 = (I_a - \frac{V_m}{Z} \sin(\alpha - \phi) + \frac{E}{R})$$

$$i(\theta) = -\frac{V_m}{Z} \sin(\pi + \alpha - \phi) - \frac{E}{R} + \{I_a - \frac{V_m}{Z} \sin(\alpha - \phi) + \frac{E}{R}\} e^{-(\theta - \frac{\pi + \alpha}{\omega})/\tau}$$

Single-Phase Full converter with Highly inductive Load



$$* V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} \sin \omega t \, d\omega t = \frac{2V_m}{\pi} \cos \alpha$$

$$\rightarrow \alpha = 90^\circ \quad \therefore V_{dc} = 0$$

$$\rightarrow \alpha < 90^\circ \quad V_{dc} = +ve$$

$$\rightarrow \alpha > 90^\circ \quad V_{dc} = -ve$$

$$I_{dc} = I_{rms} = I_a$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} (V_m \sin \omega t)^2 \, d\omega t} = \frac{V_m}{\sqrt{2}}$$

$$I_A = \frac{I_a}{2}, \quad I_R = \frac{I_a}{\sqrt{2}}$$

$$I_{s_{rms}} = I_a$$

$$I_{s_{dc}} = \text{Zero}$$

Single-Phase Dual Converter

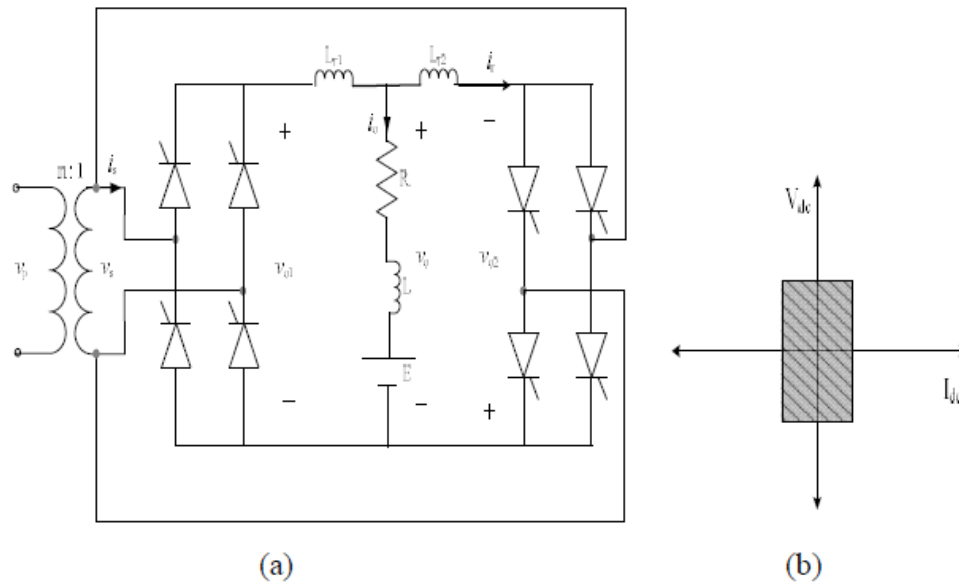


Fig. 4.14 Single phase dual converter

Used in : high power variable speed drives

$$V_{dc1} = \frac{1}{\pi} \int_{\alpha_1}^{\pi} V_m \sin \theta d\theta = \frac{2V_m}{\pi} \cos \alpha_1$$

$$V_{dc2} = \frac{1}{\pi} \int_{\alpha_2}^{\pi} V_m \sin \theta d\theta = \frac{2V_m}{\pi} \cos \alpha_2$$

Converter 1 : RECTIFIER MODE && Converter 2 : INVERTER MODE

$$V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi} \cos \alpha_1 = -\frac{2V_m}{\pi} \cos \alpha_2$$

$$\therefore \alpha_2 = 180^\circ - \alpha_1$$

phase difference between conv. 1 and conv 2 = 180°

cause : circulating current , limited by L_{r1} & L_{r2} : equal , doesn't flow through the load

circulating current advantages:

1. Both converters operate in continuous conduction mode and then independent of the load.
2. Power flows in either direction, because one converter always operates in rectifier mode while the other operates in inverter mode.

3. Since both converters operate in conduction mode, the time response for changing quadrants is fast.

circulating current disadvantages:

1. To limit the circulating current, a reactor is required and its size and cost are quite significant in high power applications.
2. The losses increased due to circulating, so that the efficiency decreased.
3. The circulating current increases the thyristors current which means a high rating thyristors are required, high cost.

Note : to remove circulating current by operating one converter at a time while the other prevent from operation by removing the firing signal from it.

- CALCULATING CIRCULATING CURRENT

For the period $\pi - \alpha < \theta < \pi + \alpha$

$$\begin{aligned}
 i_r &= \frac{1}{\omega L_r} \int_{\pi - \alpha}^{\theta} v_r d\theta = \frac{1}{\omega L_r} \int_{\pi - \alpha}^{\theta} (v_{o1} + v_{o2}) d\theta \\
 &= \frac{1}{\omega L_r} \int_{\pi - \alpha}^{\theta} 2V_m \sin \theta d\theta = \frac{2V_m}{\omega L_r} \{-\cos \theta + \cos(\pi - \alpha)\} \\
 &= -\frac{2V_m}{\omega L_r} (\cos \theta + \cos \alpha)
 \end{aligned}$$

For the periods $0 < \theta < \alpha$ and $\pi - \alpha < \theta < \pi + \alpha$

$$\begin{aligned}
 i_r &= \frac{1}{\omega L_r} \int_{2\pi - \alpha}^{\theta} v_r d\theta = \frac{1}{\omega L_r} \int_{2\pi - \alpha}^{\theta} (v_{o1} + v_{o2}) d\theta \\
 &= \frac{1}{\omega L_r} \int_{2\pi - \alpha}^{\theta} 2V_m \sin \theta d\theta = \frac{2V_m}{\omega L_r} \{\cos \theta - \cos(2\pi - \alpha)\} \\
 &= \frac{2V_m}{\omega L_r} (\cos \theta - \cos \alpha)
 \end{aligned}$$

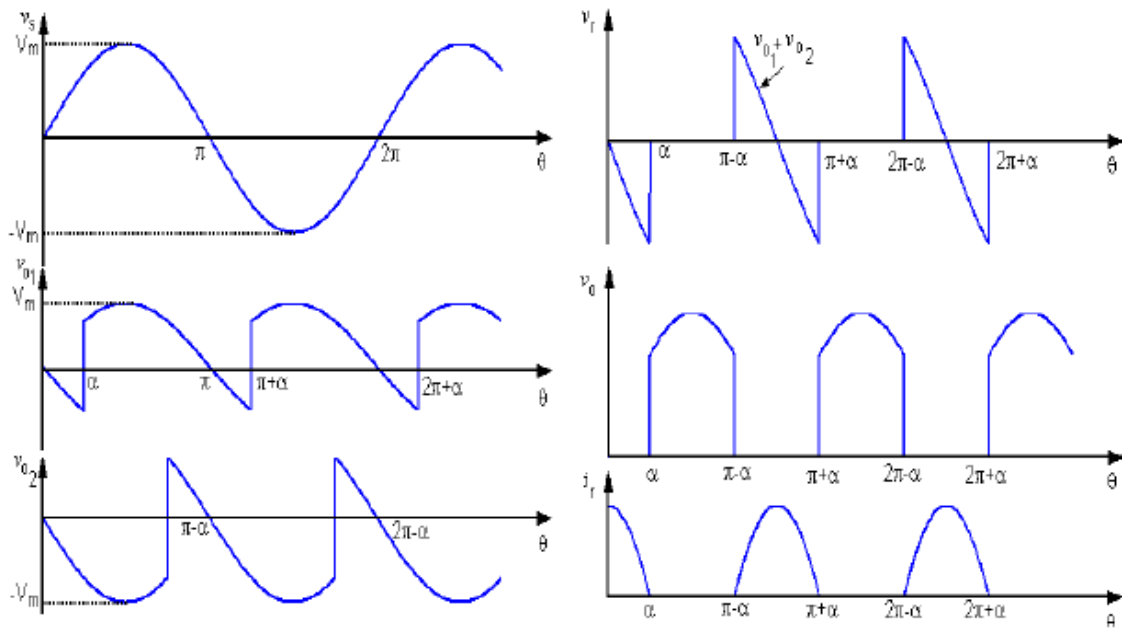
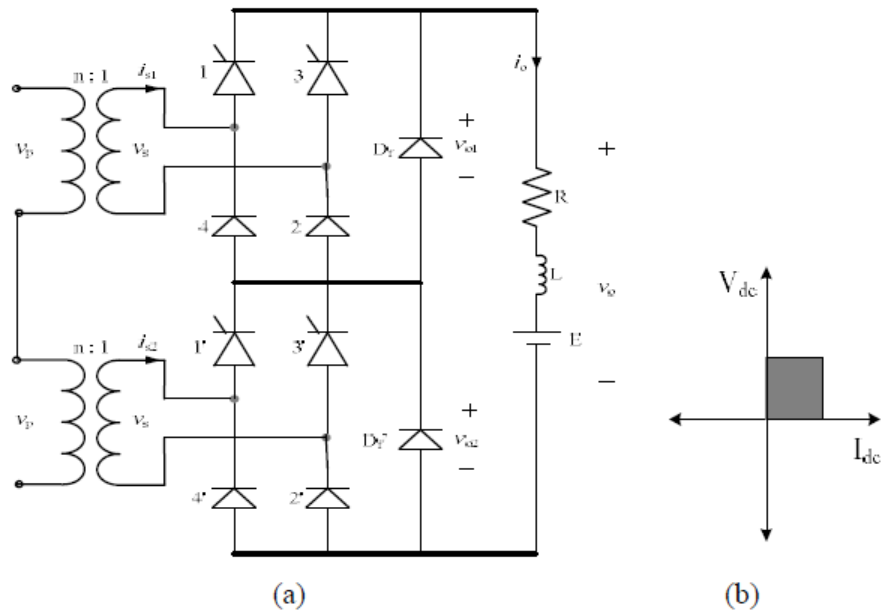


Fig. 4.15 Waveforms for single-phase dual converter.

Single-Phase Series Semi-converters

- 1- Increase control of output voltage 2-increase load voltage 3-improve power factor



$$V_{dc1} = \frac{V_m}{\pi} (\cos \alpha_1 + 1)$$

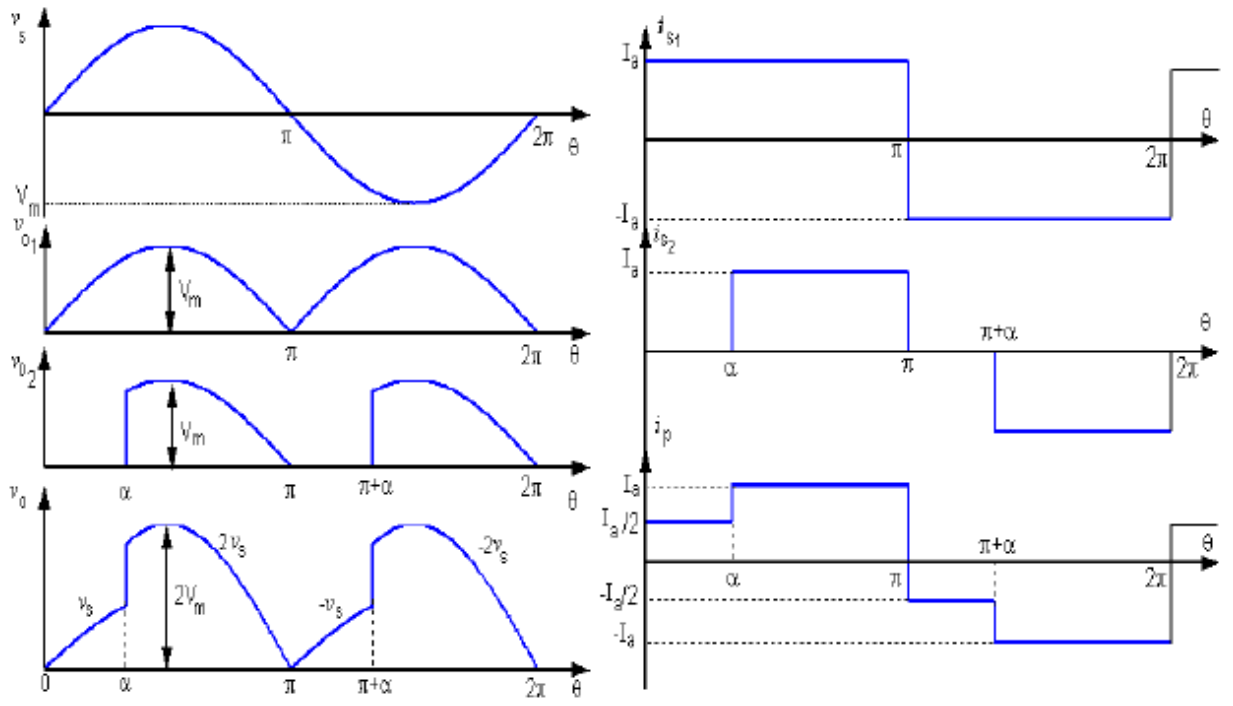
$$V_{dc2} = \frac{V_m}{\pi} (\cos \alpha_2 + 1)$$

$$V_{dc} = V_{dc1} + V_{dc2} = \frac{V_m}{\pi} (2 + \cos \alpha_1 + \cos \alpha_2)$$

$$V_{dm} = \frac{4V_m}{\pi} \text{ at } \alpha_1 = \alpha_2 = 0.$$

$$V_n = \frac{V_{odc}}{V_{odc_{max}}}$$

Mode 1 : $\alpha_1 = 0, \alpha_2 = 0: \pi$ $V_{dc} = 0.5 V_{dm} : V_{dm}$ [$V_n \geq 50\%$]



$$V_{dc1} = \frac{2V_m}{\pi}$$

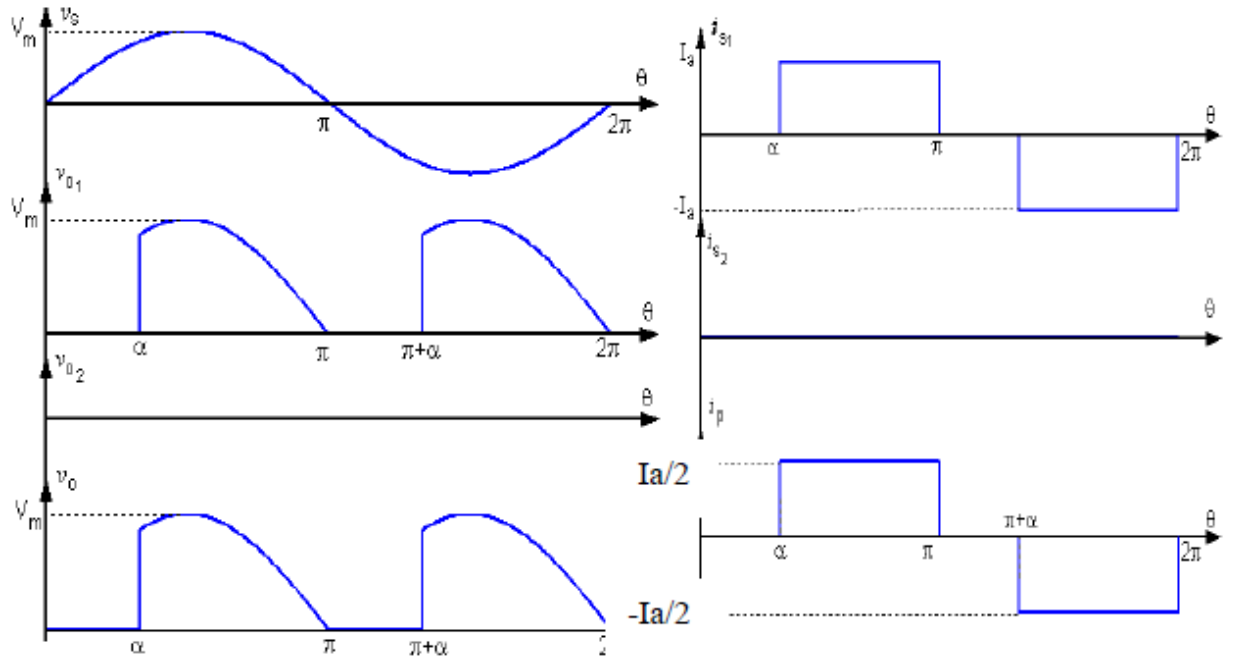
$$V_{dc2} = \frac{V_m}{\pi}(\cos \alpha_2 + 1)$$

$$V_{dc} = V_{dc1} + V_{dc2} = \frac{V_m}{\pi}(3 + \cos \alpha_2)$$

And the average output varies as $0.5 V_{dm} \leq V_{dc} < V_{dm}$

$$V_n = \frac{1}{4}(2 + 1 + \cos \alpha_2)$$

Mode 2 : $\alpha_1 = 0 : \pi$, $\alpha_2 = \pi$ (bypass by FWD) $V_{dc} = 0 : 0.5 V_{dm}$ [$V_n \leq 50\%$]



$$V_{dc1} = \frac{V_m}{\pi} (\cos \alpha_1 + 1)$$

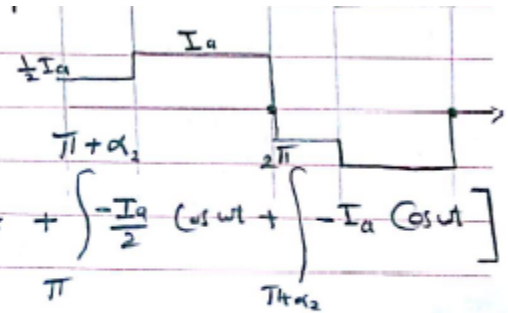
$$V_{dc2} = 0$$

$$V_{dc} = V_{dc1} + V_{dc2} = \frac{V_m}{\pi} (1 + \cos \alpha_1)$$

And the average output varies as $0 \leq V_{dc} < 0.5 V_{dm}$

$$V_n = \frac{1}{4} (2 + \cos \alpha_1 + \cos \alpha_2)$$

$$\begin{aligned}
 a_1 &= \frac{2}{T} \int_0^T i_p(\omega t) \cos \omega t \, d\omega t \\
 &= \frac{2}{2\pi} \left[\int_0^{\alpha_2} \frac{I_a}{2} \cos \omega t \, d\omega t + \int_{\alpha_2}^{\pi} I_a \cos \omega t \, d\omega t + \int_{\pi}^{\pi+\alpha_2} -\frac{I_a}{2} \cos \omega t \, d\omega t + \int_{\pi+\alpha_2}^{2\pi} -I_a \cos \omega t \, d\omega t \right] \\
 &= -\frac{I_a}{\pi} \sin \alpha
 \end{aligned}$$



$$b_1 = \frac{2}{T} \int_0^T i_p(\omega t) \sin \omega t \, d\omega t = \frac{I_a}{\pi} (\cos \alpha + 3)$$

$$\therefore I_{p1} = \sqrt{\frac{a_1^2 + b_1^2}{2}} = \frac{I_a}{\pi} \sqrt{5 + 3 \cos \alpha}$$

$$\therefore \phi_1 = \tan^{-1} \frac{a_1}{b_1} = \tan^{-1} \left(\frac{-\sin \alpha}{\cos \alpha + 3} \right)$$

$$D.F = \cos \phi_1 = \cos \left(\tan^{-1} \frac{-\sin \alpha}{3 + \cos \alpha} \right)$$

$$PF_{in} = \frac{I_{p1}}{I_p} D.F$$

$$I_p = I_a \sqrt{\left(\frac{\pi - 0.75\alpha}{\pi} \right)} \quad \leftarrow \text{Rms value of } i_p \text{ الـ H.I.L}$$

$$\therefore PF_{in} = \frac{\sqrt{5 + 3 \cos \alpha}}{\sqrt{\pi(\pi - 0.75\alpha)}} \cos \left(\tan^{-1} \frac{\sin \alpha}{3 + \cos \alpha} \right) \quad \#$$

$$\therefore HF_{in} = \sqrt{\left(\frac{I_p}{I_{p1}} \right)^2 - 1} = \sqrt{\frac{\pi(\pi - 0.75\alpha)}{5 + 3 \cos \alpha} - 1} \quad \#$$

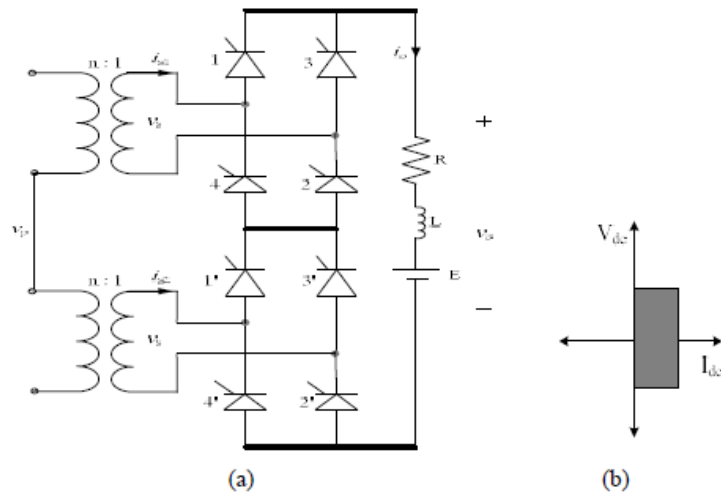
Single-Phase Series full converter

$$V_{dc1} = \frac{2V_m}{\pi} \cos \alpha_1$$

$$V_{dc2} = \frac{2V_m}{\pi} \cos \alpha_2$$

$$V_{dc} = V_{dc1} + V_{dc2} = \frac{2V_m}{\pi} (\cos \alpha_1 + \cos \alpha_2)$$

$$V_{dm} = \frac{4V_m}{\pi} \text{ at } \alpha_1 = \alpha_2 = 0.$$

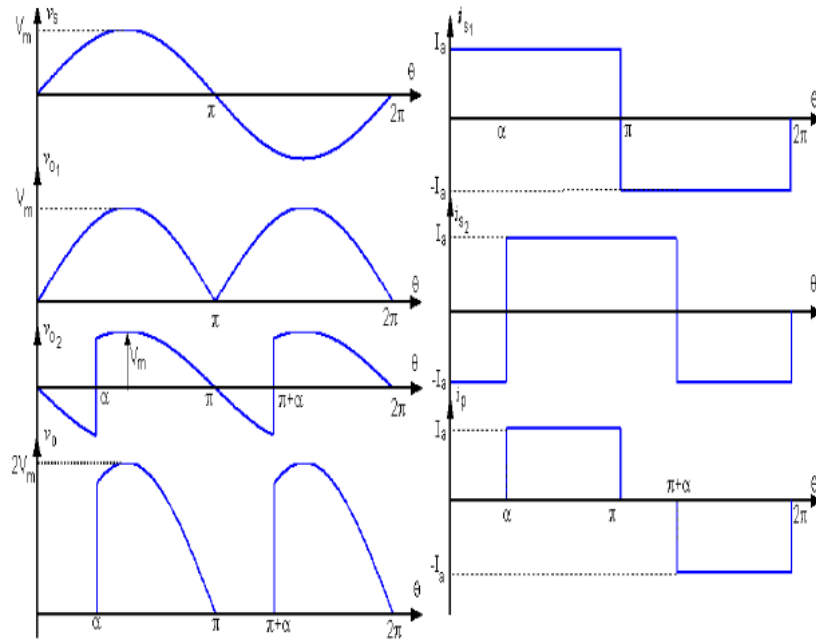


mode 1 : $\alpha_1 = 0$, $\alpha_2 = 0: \pi$ $V_n \geq 50\%$

$$V_n = \frac{1}{2} (1 + \cos \alpha_2)$$

$$V_{dc1} = \frac{2V_m}{\pi} \text{ and } V_{dc2} = \frac{2V_m}{\pi} \cos \alpha_2$$

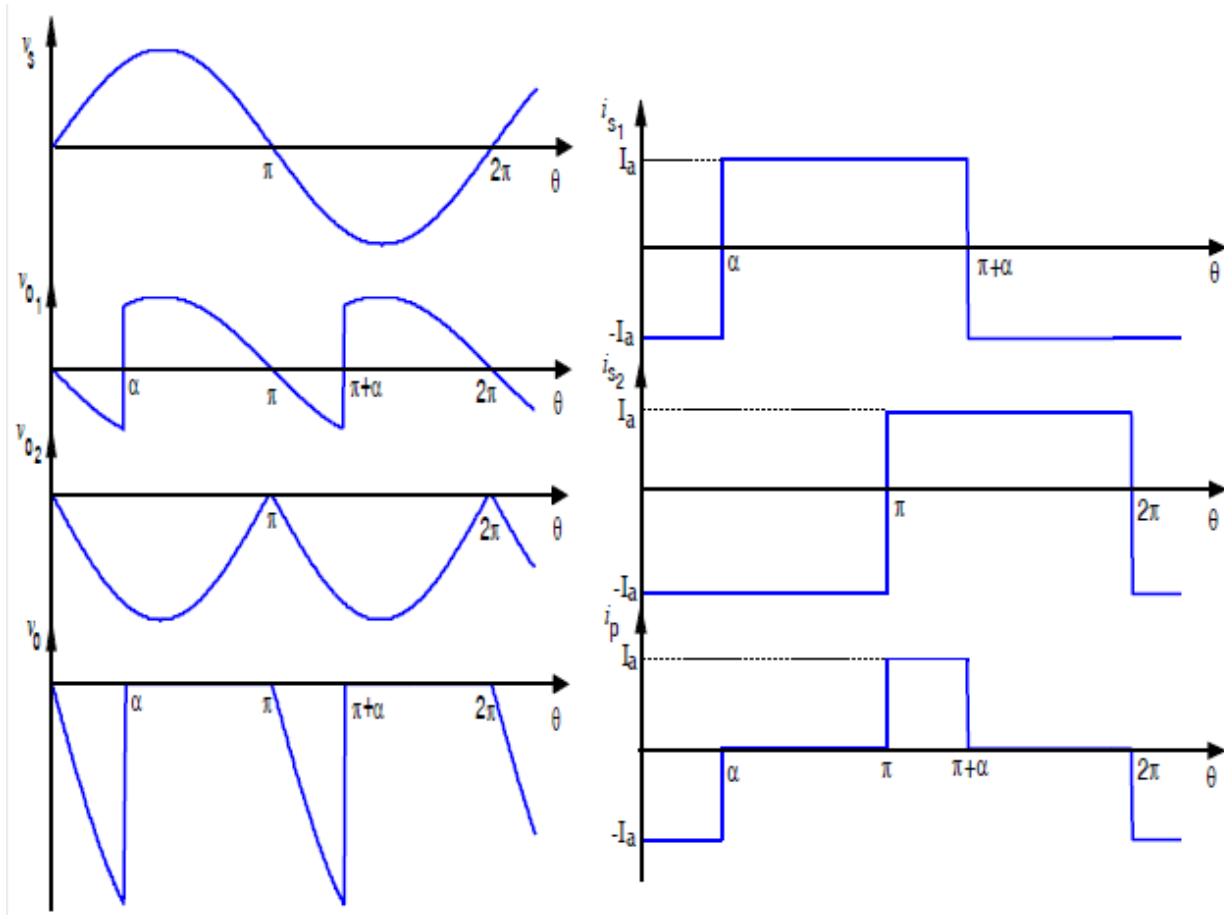
$$V_{dc} = V_{dc1} + V_{dc2} = \frac{2V_m}{\pi} (1 + \cos \alpha_2)$$



mode 2 : $\alpha_1 = 0: \pi$ $\alpha_2 = \pi$ $V_n \leq 50\%$

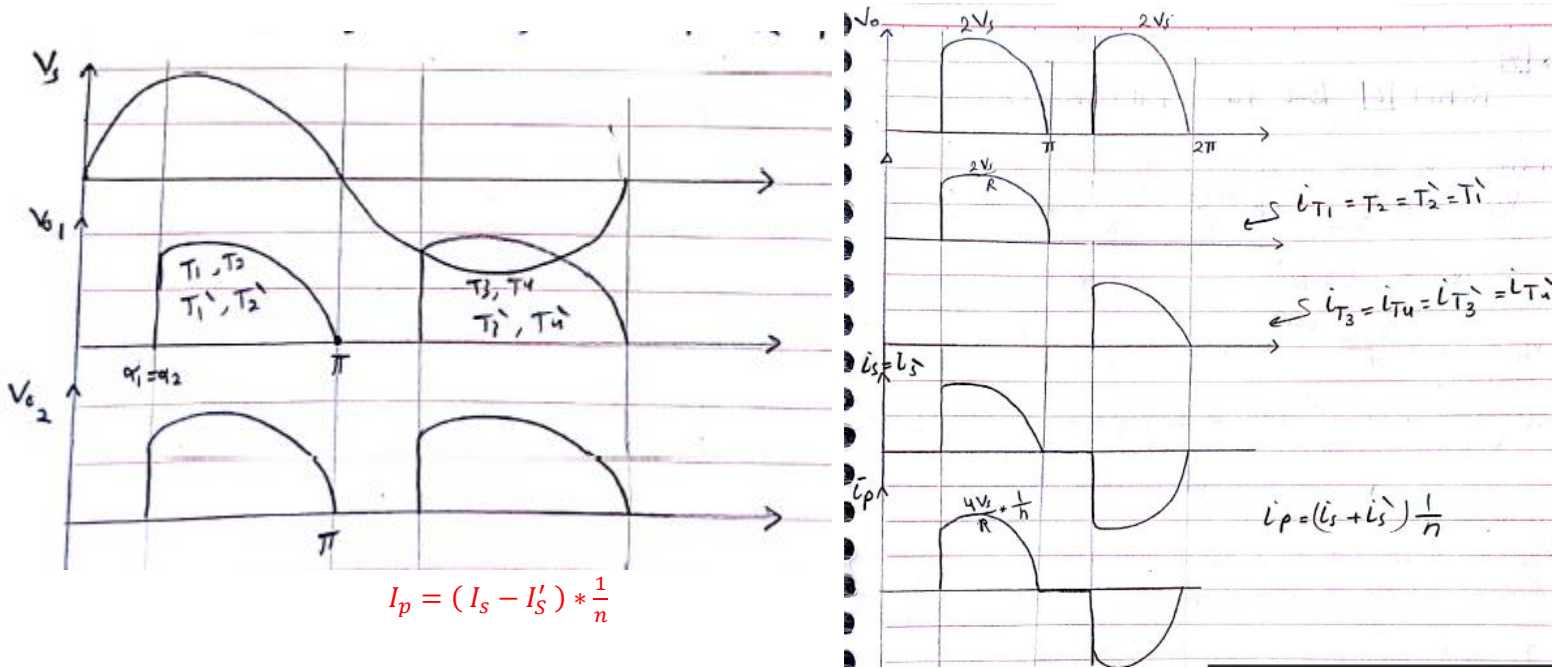
$$V_{dc1} = \frac{2V_m}{\pi} \cos \alpha_1 \text{ and } V_{dc2} = \frac{-2V_m}{\pi}$$

$$V_{dc} = V_{dc1} + V_{dc2} = \frac{2V_m}{\pi} (\cos \alpha_1 - 1)$$



R - Load

In this case must $\alpha_1 = \alpha_2$ to there be a path for the current



$$V_{o1} = \frac{V_m}{\pi} (1 + \cos \alpha_1)$$

$$V_{o2} = \frac{V_m}{\pi} (1 + \cos \alpha_2)$$

$$V_o = \frac{V_m}{\pi} (2 + \cos \alpha_1 + \cos \alpha_2)$$

$$V_o = \frac{V_m}{\pi} (2 + 2 \cos \alpha) = \frac{2V_m}{\pi} (1 + \cos \alpha)$$

Three Phase semiconverter

Used in 120 kW applications

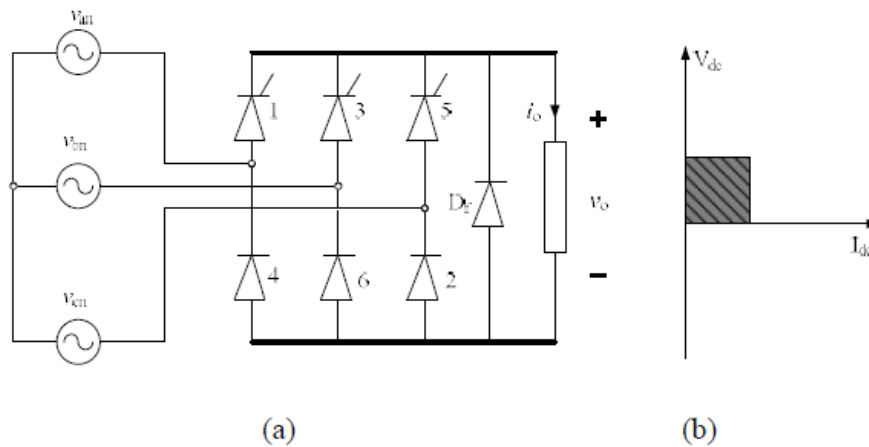


Fig. 3.21 Three-phase semiconverter circuit

upper SCR

$$T_1 \rightarrow 30 + \alpha$$

$$T_3 \rightarrow 150 + \alpha$$

$$T_5 \rightarrow 270 + \alpha$$

Lower Diodes

more negative will be on

mode 1 : $\alpha < 60$	mode 2 : $\alpha > 60$
<i>ex : $\alpha = 30$</i> $T_1 \rightarrow 30 + 30 = 60$ $T_3 \rightarrow 150 + 30 = 180$ $T_5 \rightarrow 270 + 30 = 300$ $Period = \frac{2\pi}{3}$ $V_{dc} = \frac{3}{2\pi} \left\{ \int_{30+\alpha}^{90} v_{ab} d\theta + \int_{90}^{150+\alpha} v_{ac} d\theta \right\} = \frac{3\sqrt{3}V_m}{\pi} \left\{ -\cos(\theta + 30^\circ) \Big _{30+\alpha}^{90} - \cos(\theta - 30^\circ) \Big _{90}^{150+\alpha} \right\}$ $= \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos\alpha)$ V_{dc} $= \frac{3V_m}{2} \sqrt{\frac{1}{\pi} \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}$ $V_{rms} =$	$V_{dc} = \frac{3}{2\pi} \left\{ \int_{30+\alpha}^{210} v_{ac} d\theta = \frac{3\sqrt{3}V_m}{\pi} \left\{ -\cos(\theta - 30^\circ) \right\}_{30+\alpha}^{210} \right\}$ $\} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos\alpha)$ $V_{rms} = \sqrt{\frac{3}{2\pi} \left\{ \int_{30+\alpha}^{210} v_{rms}^2 d\theta \right\}} = \sqrt{3}V_m \sqrt{\frac{3}{4\pi} \left(\pi - \alpha + \frac{1}{2} \sin 2\alpha \right)}$

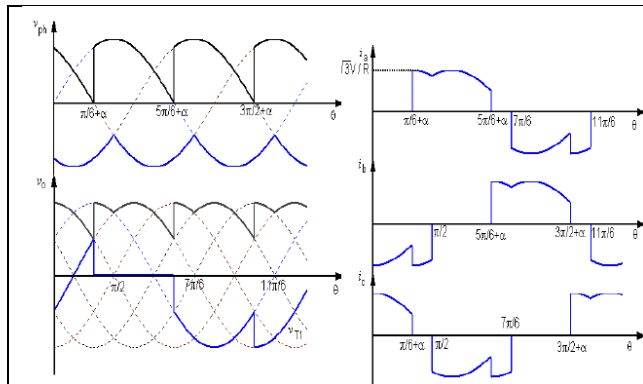


Fig. 3.22 Three phase semiconverter with resistive load and $\alpha = 30^\circ$.

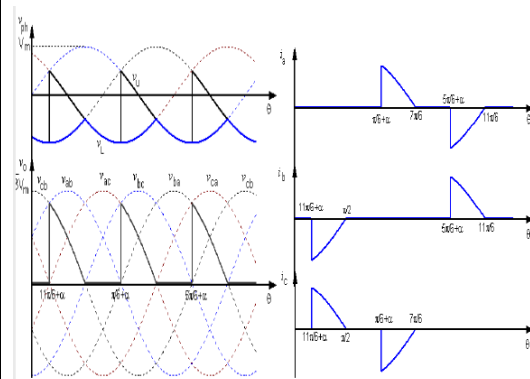


Fig. 3.24 Three phase semiconverter with resistive load and $\alpha = 120^\circ$.

$$RF = \sqrt{\left[\frac{V_{rms}}{V_{dc}}\right]^2 - 1} \quad \% \eta = \frac{V_{dc} I_{dc}}{V_{rms} I_{rms}} \times 100 \quad TUF = \frac{V_{dc} I_{dc}}{3 V_s I_s} \quad I_A = \frac{I_{dc}}{3}$$

$$I_R = \frac{I_{rms}}{\sqrt{3}} \quad P_{dc} = V_{dc} I_{dc} = P_{ac} = V_{rms} I_{rms} = S = \sqrt{3} V_L I_s \quad PF = \frac{P_o}{S} = \frac{P_{ac}}{S}$$

Three-Phase Full-Wave Controlled Rectifier

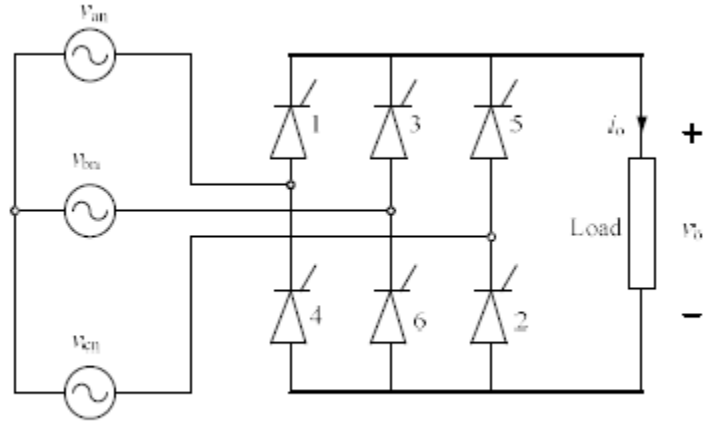


Fig. 4.26 Three-phase full converter circuit

upper SCR

$$T_1 \rightarrow 30 + \alpha$$

$$T_3 \rightarrow 150 + \alpha$$

$$T_5 \rightarrow 270 + \alpha$$

Lower SCR

$$T_4 \rightarrow 90 + \alpha$$

$$T_6 \rightarrow 210 + \alpha$$

$$T_2 \rightarrow 330 + \alpha$$

Mode 1 ($\alpha < 60^\circ$)	Mode 2 ($\alpha > 60^\circ$)
$V_{dc} = \frac{3\sqrt{3}V_m}{\pi} (\cos \alpha)$	$V_{dcm} = \frac{3\sqrt{3}V_m}{\pi}$
$V_{dcm} = \frac{3\sqrt{3}V_m}{\pi}$	$V_{rms} = 3 V_m \sqrt{\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right]}$
$V_n = \cos \alpha$	
$V_{rms} = 3 V_m \sqrt{\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right]}$	نفس القوانين فمش شرط نقسمها مودز

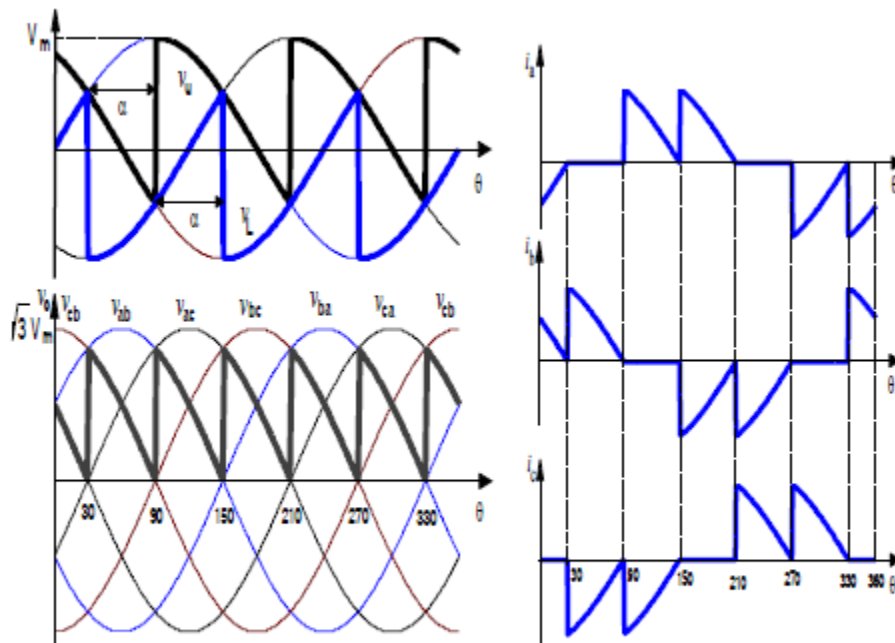


Fig. 2.27 Waveforms of output voltage, output current, and phase currents at $\alpha = 60^\circ$ for three phase full converter with resistive load.

$$V_{dc} = \frac{3}{\pi} \int_{30+\alpha}^{90+\alpha} \sqrt{3} V_m \sin(\theta + 30^\circ) d\theta = \frac{3\sqrt{3}V_m}{\pi} (-\cos(\theta + 30^\circ)) \Big|_{30+\alpha}^{90+\alpha}$$

$$= \frac{3\sqrt{3}V_m}{\pi} \cos \alpha$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{3\sqrt{3}V_m}{\pi R} \cos \alpha$$

$$I_A = \frac{1}{T} \int_0^T i_1 d\theta = 2 \times \frac{1}{2\pi} \int_{30+\alpha}^{90+\alpha} \frac{\sqrt{3}V_m}{R} \sin(\theta + 30^\circ) d\theta = \frac{1}{3} \frac{V_{dc}}{R} = \frac{I_{dc}}{3} =$$

$$= \frac{\sqrt{3}V_m}{\pi R} \cos \alpha.$$

$$P_{dc} = I_{dc} V_{dc} = \frac{27V_m^2}{\pi^2 R} \cos^2 \alpha = 2.736 \frac{V_m^2}{R} \cos^2 \alpha$$

$$\begin{aligned}
 V_{rms} &= \sqrt{\frac{3}{\pi} \int_{30+\alpha}^{90+\alpha} v_{ab}^2 d\theta} = \sqrt{\frac{3}{\pi} \int_{30+\alpha}^{90+\alpha} 3V_m^2 \sin^2(\theta + 30^\circ) d\theta} \\
 &= \sqrt{\frac{9V_m^2}{2\pi} \left[\theta - \frac{1}{2} \sin(2\theta + 60^\circ) \right]_{30+\alpha}^{90+\alpha}} = \sqrt{\frac{9V_m^2}{2\pi} \left[\frac{\pi}{3} - \frac{1}{2} [\sin(240^\circ + 2\alpha) - \sin(120^\circ + 2\alpha)] \right]} \\
 &= 3V_m \sqrt{\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right]}
 \end{aligned}$$

$$I_R = \frac{I_{rms}}{\sqrt{3}} = \frac{\sqrt{3}V_m \sqrt{\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right]}}{R}$$

AC output power is

$$I_s = \sqrt{2}I_R = \frac{\sqrt{6}V_m \sqrt{\frac{1}{2\pi} \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}{R}$$

$$P_{ac} = V_{rms} I_{rms} = \frac{9V_m^2 \left(\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right] \right)}{R}$$

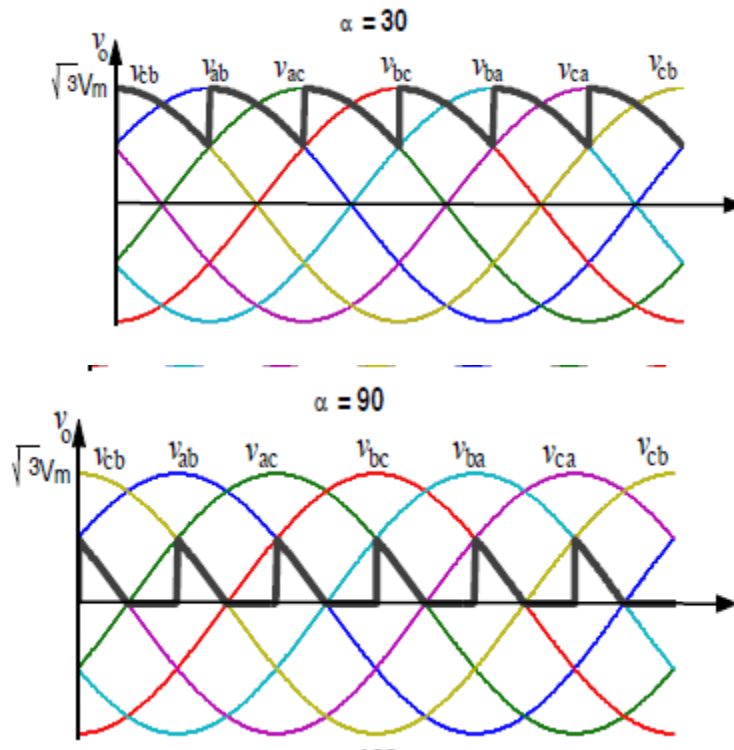
$$\% \eta = \frac{P_{dc}}{P_{ac}} \times 100 = \frac{\frac{27V_m^2}{\pi^2 R} \cos^2 \alpha}{\frac{9V_m^2 \left(\frac{1}{2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \cos 2\alpha \right] \right)}{R}} = \frac{6}{\pi} \frac{\cos^2 \alpha}{\left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)} \times 100$$

$$FF = \frac{V_{rms}}{V_{dc}} = \frac{\pi \times \sqrt{\frac{1}{2\pi} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}{\sqrt{3} \cos \alpha}$$

$$RF = \sqrt{FF^2 - 1} = \sqrt{\frac{\frac{\pi}{2} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}{3 \cos^2 \alpha} - 1}$$

$$S = 3V_s I_s = 3 \times \frac{V_m}{\sqrt{2}} \times \frac{\sqrt{6}V_m \sqrt{\frac{1}{2\pi} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}{R} = \frac{3\sqrt{3}V_m^2 \sqrt{\frac{1}{2\pi} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}{R}$$

$$TUF = \frac{\frac{27V_m^2}{\pi^2 R} \cos^2 \alpha}{\frac{3\sqrt{3}V_m^2 \sqrt{\frac{1}{2\pi} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}{R}} = \frac{3\sqrt{3} \cos^2 \alpha}{\pi^2 \sqrt{\frac{1}{2\pi} \left(\frac{3}{2} + \frac{\sqrt{3}}{2} \cos 2\alpha \right)}}$$



Three-Phase Full Converter with Commutation

μ commutation or overlap angle because of source inductance 2 SCRs will conduct

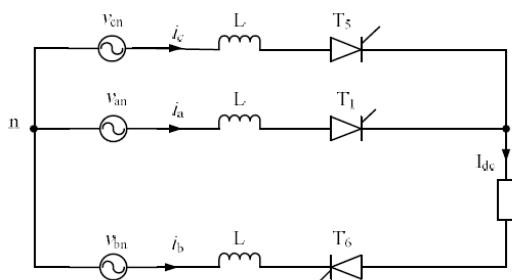


Fig. (4.30) equivalent circuit for conducting transfer from D_5 and D_1 .

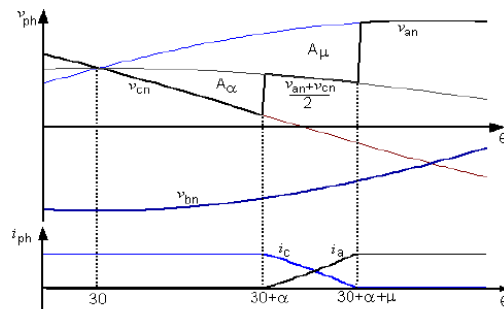


Fig. (4.31) Reduction in output voltage when conduction transfer from phase c to phase a (or conduction transfer from T_5 to T_1)

The potential at node x is:

$$v_x = v_{an} - L \frac{di_a}{dt}$$

$$v_x = \frac{v_{an} + v_{cn}}{2}$$

Reduction area

$$A = \int_{30+\alpha}^{30+\alpha+\mu} (v_x - v_{an}) d\omega t = \int_{30+\alpha}^{30+\alpha+\mu} \frac{v_{an} - v_{cn}}{2} d\omega t$$

$$= \int_0^{I_{dc}} \omega L \frac{di}{dt} dt = \int_0^{I_{dc}} \omega L di = \omega L I_{dc}$$

Reduction output voltage (repeat every $\frac{\pi}{3}$)

$$\Delta V_{dc} = \frac{\omega L I_{dc}}{\pi/3} = \frac{3 L I_{dc} (2\pi f)}{\pi} = 6 f L I_{dc}$$

Reduction of output voltage

$$V_{dc} = V_{dco} - \Delta V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha - 6 f L I_{dc}$$

Phase current during commutation

$$i_b = -I_{dc}$$

$$\therefore \frac{di_a}{dt} = \frac{v_{an} - v_{cn}}{2L} \quad \text{and} \quad \therefore \int_0^{i_a} di_a = \int_{30}^{\omega t} \frac{v_{an} - v_{cn}}{2\omega L} d\omega t$$

$$i_a = \int_{\alpha+30}^{\omega t} \frac{\sqrt{3}V_m \sin(\omega t - 30)}{2\omega L} d\omega t = \frac{\sqrt{3}V_m}{2\omega L} [\cos \alpha - \cos(\omega t - 30)]$$

$$i_c = I_{dc} - i_a = I_{dc} - \frac{\sqrt{3}V_m}{2\omega L} [\cos \alpha - \cos(\omega t - 30)]$$

Commutation angle put $\omega = 30 + \alpha + \mu$

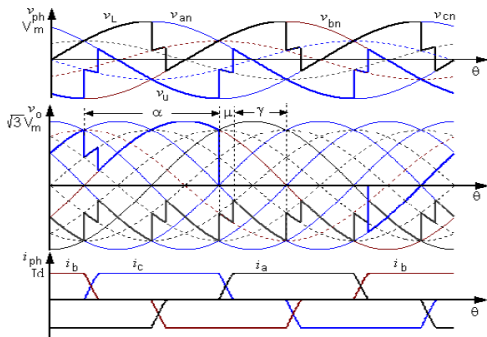


Fig. (4.33) Three-phase full converter with commutation and $\alpha = 120^\circ$

$$I_{dc} = \frac{\sqrt{3}V_m}{2\omega L} \{ \cos \alpha - \cos(\alpha + \mu) \}$$

$$\therefore \mu = -\alpha + \cos^{-1} \left(\cos \alpha - \frac{2\omega L I_{dc}}{\sqrt{3}V_m} \right)$$

$$\therefore \mu = -\alpha + \cos^{-1} \left(\cos \alpha - \frac{2\omega L I_{dc}}{\sqrt{3}V_m} \right)$$

Table (4.5) Phase currents during all intervals in three-phase full-wave bridge controlled rectifier.

Interval		Phase currents		
From	To	i_a	i_b	i_c
0	$\pi/6+\alpha$	0	$-I_{dc}$	I_{dc}
$\pi/6$	$\pi/6+\alpha+\mu$	$\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 30^\circ)]$	$-I_{dc}$	$I_{dc} - \frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 30^\circ)]$
$\pi/6+\alpha+\mu$	$\pi/2+\alpha$	I_{dc}	$-I_{dc}$	0
$\pi/2+\alpha$	$\pi/2+\alpha+\mu$	I_{dc}	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 90^\circ)]$	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 90^\circ)]$
$\pi/2+\alpha+\mu$	$5\pi/6+\alpha$	I_{dc}	0	$-I_{dc}$
$5\pi/6+\alpha$	$5\pi/6+\alpha+\mu$	$I_{dc} - \frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 150^\circ)]$	$\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 150^\circ)]$	$-I_{dc}$
$5\pi/6+\alpha+\mu$	$7\pi/6+\alpha$	0	I_{dc}	$-I_{dc}$
$7\pi/6+\alpha$	$7\pi/6+\alpha+\mu$	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 210^\circ)]$	I_{dc}	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 210^\circ)]$
$7\pi/6+\alpha+\mu$	$9\pi/6+\alpha$	$-I_{dc}$	I_{dc}	0
$9\pi/6+\alpha$	$9\pi/6+\alpha+\mu$	$-I_{dc}$	$I_{dc} - \frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 270^\circ)]$	$\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 270^\circ)]$
$9\pi/6+\alpha+\mu$	$11\pi/6+\alpha$	$-I_{dc}$	0	I_{dc}
$11\pi/6$	$11\pi/6+\mu$	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 330^\circ)]$	$-\frac{\sqrt{3}V_m}{2mL}[\cos\alpha - \cos(\omega t - 330^\circ)]$	I_{dc}
$11\pi/6+\mu$	2π	0	$-I_{dc}$	I_{dc}

Power Factor Improvement

The power factor improvement can be implemented with **forced commutation technique** by gate turn off thyristors

methods are:

1. Extinction angle control method.
2. Symmetrical angle control method.
3. Pulse-width modulation method.(PWM)
4. Sinusoidal pulse-width modulation method.(SPWM)

4.9.1 Extinction angle control method

With : Both single phase full converter and semiconverter S_1 : on @ $\theta = 0$ off @ $\theta = \pi - \beta$
 S_3 : on @ $\theta = \pi$ off @ $\theta = 2\pi - \beta$

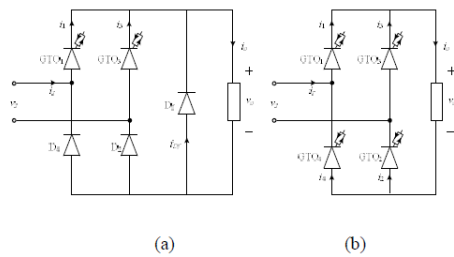
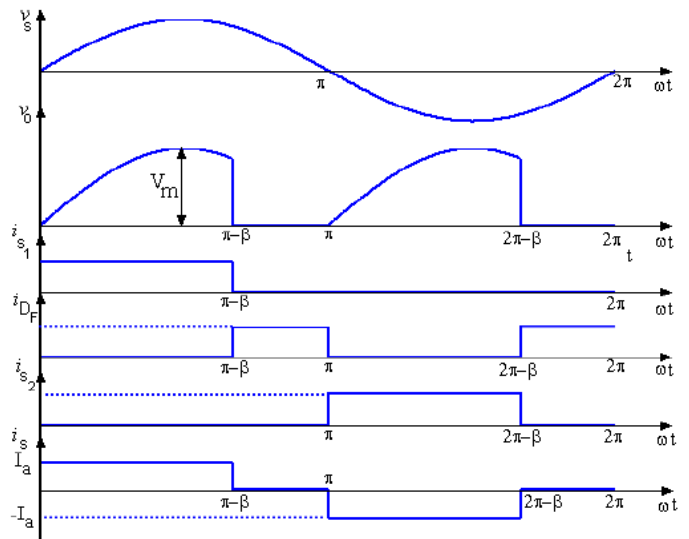


Fig. 4.38 Extinction angle control circuits

$$V_{dc} = \frac{1}{\pi} \int_0^{\pi-\beta} v_o d\theta = \frac{1}{\pi} \int_0^{\pi-\beta} V_m \sin \theta d\theta = \frac{V_m}{\pi} (1 + \cos \beta)$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi-\beta} v_o^2 d\theta} = \sqrt{\frac{1}{\pi} \int_0^{\pi-\beta} V_m^2 \sin^2 \theta d\theta} = V_m \sqrt{\frac{1}{2\pi} (\pi - \beta + \frac{1}{2} \sin 2\beta)}$$



The fundamental component of secondary side current is **leading** the secondary side voltage of converter transformer so that the input power factor

Example 4.12

A single phase semiconverter is used for extinction angle control. If the load current is continuous with negligible ripple, determine the input power factor, the input harmonic factor and transformer utilization factor.

Solution:

The waveforms of output voltage and secondary side current are illustrated in Fig. 4.39.

$$\begin{aligned}
 a_1 &= \frac{1}{\pi} \left\{ \int_0^{\pi-\beta} I_a \cos \theta \, d\theta - \int_{\pi}^{2\pi-\beta} I_a \cos \theta \, d\theta \right\} = \frac{I_a}{\pi} \{ \sin \theta \Big|_0^{\pi-\beta} - \sin \theta \Big|_{\pi}^{2\pi-\beta} \} \\
 &= \frac{I_a}{\pi} \{ \sin(\pi - \beta) - \sin 0 - \sin(2\pi - \beta) + \sin \pi \} = \frac{2I_a}{\pi} \sin \beta \\
 b_1 &= \frac{1}{\pi} \left\{ \int_0^{\pi-\beta} I_a \sin \theta \, d\theta - \int_{\pi}^{2\pi-\beta} I_a \sin \theta \, d\theta \right\} = \frac{I_a}{\pi} \{ -\cos \theta \Big|_0^{\pi-\beta} + \cos \theta \Big|_{\pi}^{2\pi-\beta} \} \\
 &= \frac{I_a}{\pi} \{ -\cos(\pi - \beta) + \cos 0 + \cos(2\pi - \beta) - \cos \pi \} = \frac{2I_a}{\pi} (1 + \cos \beta) \\
 c_1 &= \sqrt{a_1^2 + b_1^2} = \frac{2I_a}{\pi} \sqrt{\sin^2 \beta + (1 + \cos \beta)^2} = \frac{4I_a}{\pi} \cos \frac{\beta}{2} \\
 I_{s1} &= \frac{c_1}{\sqrt{2}} = \frac{4I_a}{\sqrt{2}\pi} \cos \frac{\beta}{2} = \frac{2\sqrt{2}I_a}{\pi} \cos \frac{\beta}{2}
 \end{aligned}$$

The angle of fundamental component of input current is:

$$\phi_1 = \tan^{-1} \frac{a_1}{b_1} = \tan^{-1} \frac{\sin \beta}{1 + \cos \beta} = \tan^{-1} \frac{(2 \sin \frac{\beta}{2} \cos \frac{\beta}{2})}{(2 \cos \frac{\beta}{2} \cos \frac{\beta}{2})} = \frac{\beta}{2}$$

The sign of ϕ_1 is positive which means the power factor is leading.

The displacement factor is:

$$Df = \cos \phi_1 = \cos \frac{\beta}{2}$$

The rms value of secondary side current is:

$$I_s = \sqrt{\frac{I_a^2 (\pi - \beta) \times 2}{2\pi}} = I_a \sqrt{\frac{\pi - \beta}{\pi}}$$

The input power factor is:

$$PF = \frac{I_{s1}}{I_s} Df = \frac{\left(\frac{2\sqrt{2}I_a \cos \frac{\beta}{2}}{\pi} \right) \cos \frac{\beta}{2}}{\left(I_a \sqrt{\frac{\pi - \beta}{\pi}} \right)} = \frac{2\sqrt{2}}{\sqrt{\pi(\pi - \beta)}} \cos^2 \frac{\beta}{2} = \frac{\sqrt{2}}{\sqrt{\pi(\pi - \beta)}} (1 + \cos \beta)$$

The harmonic factor of input current is:

$$HF = \sqrt{\left(\frac{I_s}{I_{s1}} \right)^2 - 1} = \sqrt{\left[\frac{I_a \sqrt{\frac{\pi - \beta}{\pi}}}{\left(\frac{2\sqrt{2}I_a \cos \frac{\beta}{2}}{\pi} \right)} \right]^2 - 1} = \sqrt{\frac{\pi(\pi - \beta)}{8 \cos^2 \frac{\beta}{2}}} - 1 = \sqrt{\frac{\pi(\pi - \beta)}{4(1 + \cos \beta)}} - 1$$

The transformer utilization factor is:

$$TUF = \frac{P_{dc}}{S} = \frac{P_o}{S} = PF = \frac{\sqrt{2}}{\sqrt{\pi(\pi - \beta)}} (1 + \cos \beta)$$

Lead PF : means reactive power will go from circuit to the supply , if there is another lag loads the PF imp

4.9.2 Symmetrical angle control method

$$S_1 \text{ on @ } \theta = \frac{\pi - \beta}{2} \text{ off @ } \theta = \frac{\pi + \beta}{2}$$

$$S_2 \text{ on @ } \theta = \frac{3\pi - \beta}{2} \text{ off @ } \frac{3\pi + \beta}{2}$$

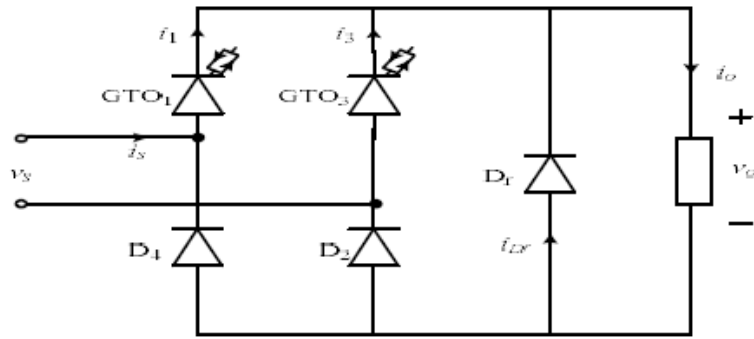


Fig. 4.40 Symmetrical angle control circuits

$$V_{dc} = \frac{1}{\pi} \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} V_m \sin \theta d\theta = \frac{V_m}{\pi} [-\cos(\frac{\pi+\beta}{2}) + \cos(\frac{\pi-\beta}{2})]$$

$$= \frac{V_m}{\pi} [-\cos \frac{\beta}{2} \cos \frac{\pi}{2} + \sin \frac{\beta}{2} \sin \frac{\pi}{2} + \cos \frac{\beta}{2} \cos \frac{\pi}{2} + \sin \frac{\beta}{2} \sin \frac{\pi}{2}] = \frac{2V_m}{\pi} \sin \frac{\beta}{2}$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} V_m^2 \sin^2 \theta d\theta} = V_m \sqrt{\frac{1}{2\pi} [\frac{\pi+\beta}{2} - \frac{\pi-\beta}{2} - \frac{1}{2} \sin(\pi+\beta) + \frac{1}{2} \sin(\pi-\beta)]}$$

$$= V_m \sqrt{\frac{1}{2\pi} (\beta - \frac{1}{2} \sin \beta \cos \pi + \frac{1}{2} \sin \beta \cos \pi)} = V_m \sqrt{\frac{1}{2\pi} (\beta + \sin \beta)}$$

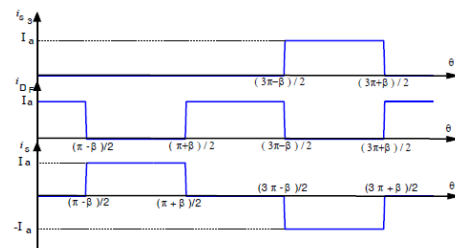
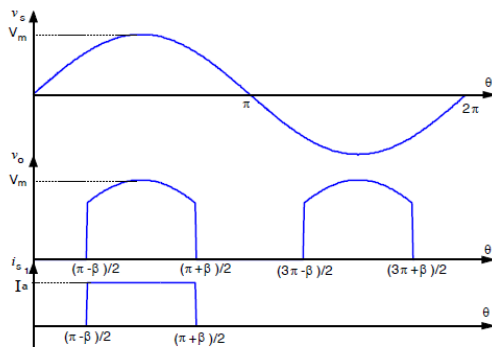


Fig. 4.38 Waveforms of extinction angle control circuits

Example 4.13

A single phase semiconverter is used for symmetrical angle control. If the load current is continuous with negligible ripple, determine the input power factor, the input harmonic factor and transformer utilization factor.

Solution:

The waveforms of output voltage and secondary side current are illustrated in Fig. 4.39.

$$\begin{aligned}
 a_1 &= \frac{1}{\pi} \left\{ \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} I_a \cos \theta \, d\theta - \int_{(3\pi-\beta)/2}^{(3\pi+\beta)/2} I_a \cos \theta \, d\theta \right\} = \frac{I_a}{\pi} \left\{ \sin \theta \Big|_{(\pi-\beta)/2}^{(\pi+\beta)/2} - \sin \theta \Big|_{(3\pi-\beta)/2}^{(3\pi+\beta)/2} \right\} \\
 &= \frac{I_a}{\pi} \left\{ \sin\left(\frac{\pi+\beta}{2}\right) - \sin\left(\frac{\pi-\beta}{2}\right) - \sin\left(\frac{3\pi+\beta}{2}\right) + \sin\left(\frac{3\pi-\beta}{2}\right) \right\} = 0 \\
 b_1 &= \frac{1}{\pi} \left\{ \int_{(\pi-\beta)/2}^{(\pi+\beta)/2} I_a \sin \theta \, d\theta - \int_{(3\pi-\beta)/2}^{(3\pi+\beta)/2} I_a \sin \theta \, d\theta \right\} = \frac{I_a}{\pi} \left\{ \cos \theta \Big|_{(\pi-\beta)/2}^{(\pi+\beta)/2} - \cos \theta \Big|_{(3\pi-\beta)/2}^{(3\pi+\beta)/2} \right\} \\
 &= \frac{I_a}{\pi} \left\{ -\cos\left(\frac{\pi+\beta}{2}\right) + \cos\left(\frac{\pi-\beta}{2}\right) + \cos\left(\frac{3\pi+\beta}{2}\right) - \cos\left(\frac{3\pi-\beta}{2}\right) \right\} = \frac{4I_a}{\pi} \sin \frac{\beta}{2} \\
 c_1 &= \sqrt{a_1^2 + b_1^2} = \frac{4I_a}{\pi} \sin \frac{\beta}{2} \\
 I_{s1} &= \frac{c_1}{\sqrt{2}} = \frac{4I_a}{\sqrt{2}\pi} \sin \frac{\beta}{2} = \frac{2\sqrt{2}I_a}{\pi} \sin \frac{\beta}{2}
 \end{aligned}$$

The angle of fundamental component of input current is:

$$\phi_1 = \tan^{-1} \frac{a_1}{b_1} = 0$$

The displacement factor is: $Df = \cos \phi_1 = 1$

The rms value of secondary side current is:

$$I_s = \sqrt{\frac{I_a^2 \times \beta \times 2}{2\pi}} = I_a \sqrt{\frac{\beta}{\pi}}$$

The input power factor is:

$$PF = \frac{I_{s1}}{I_s} Df = \frac{\left(\frac{2\sqrt{2}I_a \sin \frac{\beta}{2}}{\pi} \right)}{\left(I_a \sqrt{\frac{\beta}{\pi}} \right)} \times 1 = \frac{2\sqrt{2}}{\sqrt{\pi\beta}} \sin \frac{\beta}{2}$$

The harmonic factor of input current is:

$$HF = \sqrt{\left(\frac{I_s}{I_{s1}} \right)^2 - 1} = \sqrt{\left[\frac{I_a \sqrt{\frac{\beta}{\pi}}}{\left(\frac{2\sqrt{2}I_a \sin \frac{\beta}{2}}{\pi} \right)} \right]^2 - 1} = \sqrt{\frac{\pi\beta}{8 \sin^2 \frac{\beta}{2}} - 1}$$

The transformer utilization factor is:

$$TUF = \frac{P_o}{S} = \frac{P_o}{S} = PF = \frac{2\sqrt{2}}{\sqrt{\pi\beta}} \sin \frac{\beta}{2}$$

4.9.3 Pulse-width modulation method

triangle ,refrence V_r
square , carrier V_c

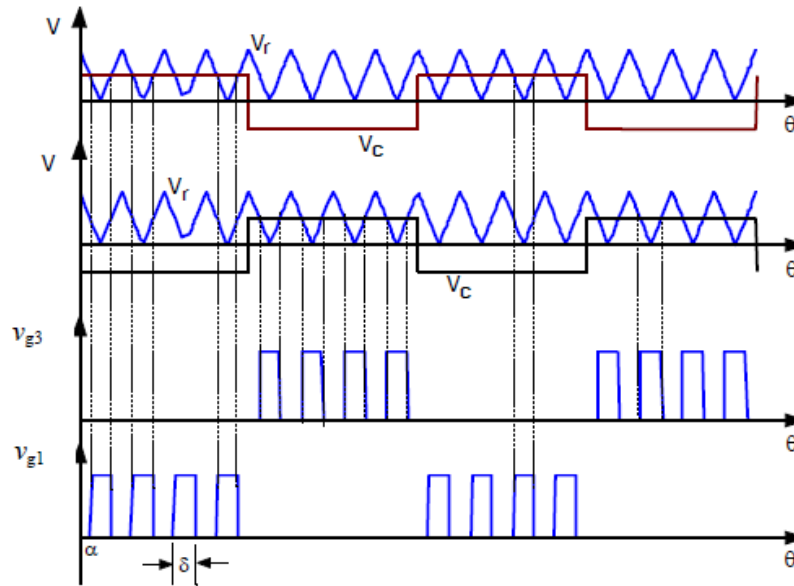


Fig. 4.42 Pulse-width modulation control.

$$V_{dc} = \sum_{k=1}^n \frac{1}{\pi} \int_{\alpha_k}^{\alpha_k + \delta_k} V_m \sin \theta d\theta = \frac{V_m}{\pi} \sum_{k=1}^n [\cos \alpha_k - \cos(\alpha_k + \delta_k)]$$

$$V_{rms} = \sqrt{\sum_{k=1}^n \frac{1}{2\pi} \int_{\alpha_k}^{\alpha_k + \delta_k} V_m^2 \sin^2 \theta d\theta} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \sum_{k=1}^n [\delta_k + \frac{1}{2} \sin 2\alpha_k - \frac{1}{2} \sin(2\alpha_k + 2\delta_k)]}$$

Where: n is the total number of pulses per each of input voltage half cycle.

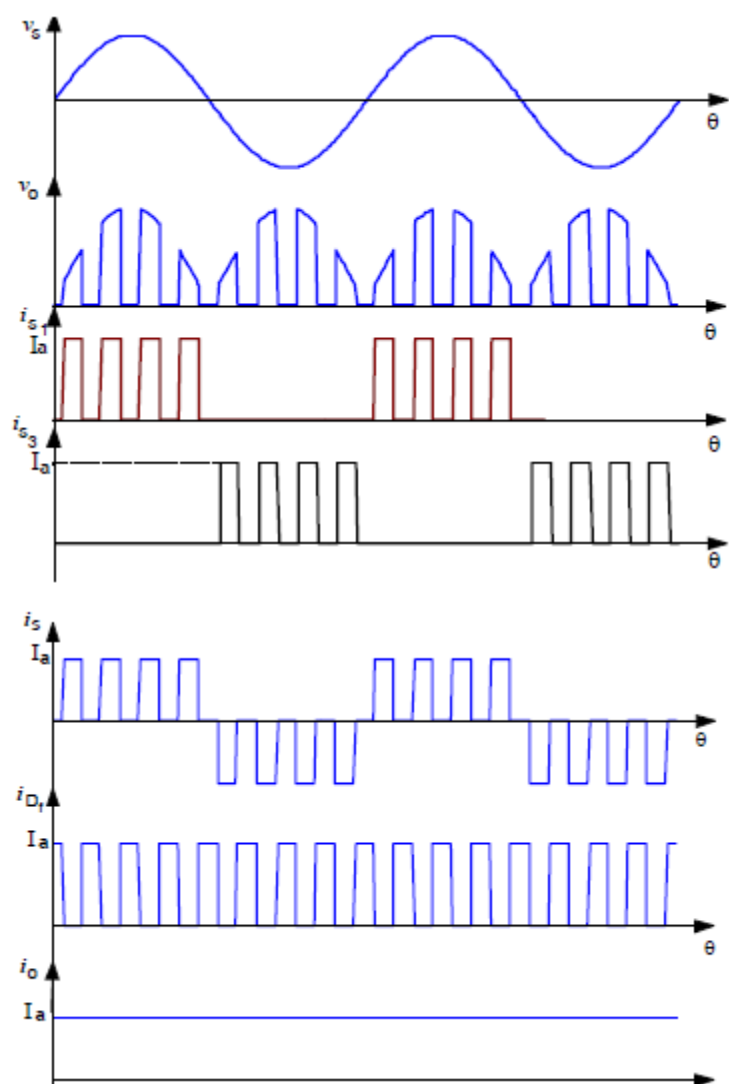
α_k and δ_k is the delay angle and width of pulse number k respectively.

$$n = x^y$$

n : possible number of switches

x : the possible number of switches per leg

y : number of legs (phases) of the converter



Example 4.14

A single phase semiconverter is used for pulse width modulation. If the load current is continuous with negligible ripple, determine the input power factor, the input harmonic factor and transformer utilization factor.

Solution:

The waveforms of output voltage and secondary side current is illustrated in Fig. 4.39.

$$a_1 = \sum_{k=1}^n \frac{1}{\pi} \left\{ \int_{\alpha_k}^{\alpha_k + \delta_k} I_a \cos \theta \, d\theta - \int_{\pi + \alpha_k}^{\pi + \alpha_k + \delta_k} I_a \cos \theta \, d\theta \right\} = 0$$

$$\begin{aligned} b_1 &= \sum_{k=1}^n \frac{1}{\pi} \left\{ \int_{\alpha_k}^{\alpha_k + \delta_k} I_a \sin \theta \, d\theta - \int_{\pi + \alpha_k}^{\pi + \alpha_k + \delta_k} I_a \sin \theta \, d\theta \right\} = \sum_{k=1}^n \frac{I_a}{\pi} \left\{ -\cos \theta \Big|_{\alpha_k}^{\alpha_k + \delta_k} + \cos \theta \Big|_{\pi + \alpha_k}^{\pi + \alpha_k + \delta_k} \right\} \\ &= \sum_{k=1}^n \frac{I_a}{\pi} \{ \cos \alpha_k - \cos(\alpha_k + \delta_k) + \cos(\pi + \alpha_k + \delta_k) - \cos(\pi + \alpha_k) \} \\ &= \frac{2I_a}{\pi} \sum_{k=1}^n \frac{I_a}{\pi} \{ \cos \alpha_k - \cos(\alpha_k + \delta_k) \} \end{aligned}$$

$$c_1 = \sqrt{a_1^2 + b_1^2} = b_1$$

$$I_{s1} = \frac{c_1}{\sqrt{2}} = \frac{\sqrt{2}I_a}{\pi} \sum_{k=1}^n \{ \cos \alpha_k - \cos(\alpha_k + \delta_k) \}$$

The angle of fundamental component of input current is:

$$\phi_1 = \tan^{-1} \frac{a_1}{b_1} = 0$$

The displacement factor is:

$$Df = \cos \phi_1 = 1$$

The rms value of secondary side current is:

$$I_s = \sqrt{\frac{I_a^2 \times \delta \times n \times 2}{2\pi}} = I_a \sqrt{\frac{\delta n}{\pi}}$$

The input power factor is:

$$PF = \frac{I_{s1}}{I_s} Df = \frac{(\frac{\sqrt{2}I_a}{\pi} \sum_{k=1}^n \{\cos\alpha_k - \cos(\alpha_k + \delta_k)\})}{(I_a \sqrt{\frac{\delta n}{\pi}})} \times 1 = \sqrt{\frac{2}{n\delta\pi} \sum_{k=1}^n \{\cos\alpha_k - \cos(\alpha_k + \delta_k)\}}$$

The harmonic factor of input current is:

$$HF = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} = \sqrt{\frac{n\delta\pi}{2} \left[\frac{1}{\sum_{k=1}^n [\cos\alpha_k - \cos(\alpha_k + \delta_k)]^2} - 1 \right]}$$

The transformer utilization factor is:

$$TUF = \frac{P_{dc}}{S} = \frac{P_o}{S} = PF = \sqrt{\frac{2}{n\delta\pi} \sum_{k=1}^n \{\cos\alpha_k - \cos(\alpha_k + \delta_k)\}}$$

4.9.4 Sinusoidal Pulse-Width Modulation Method

To control width of pulse compare it to sin wave

$$m = \frac{A_c}{A_r} = \frac{\text{peak of carrier}}{\text{peak of reference}} \quad \text{modulation index}$$

Triangle V_r reference

sin. V_c carrier

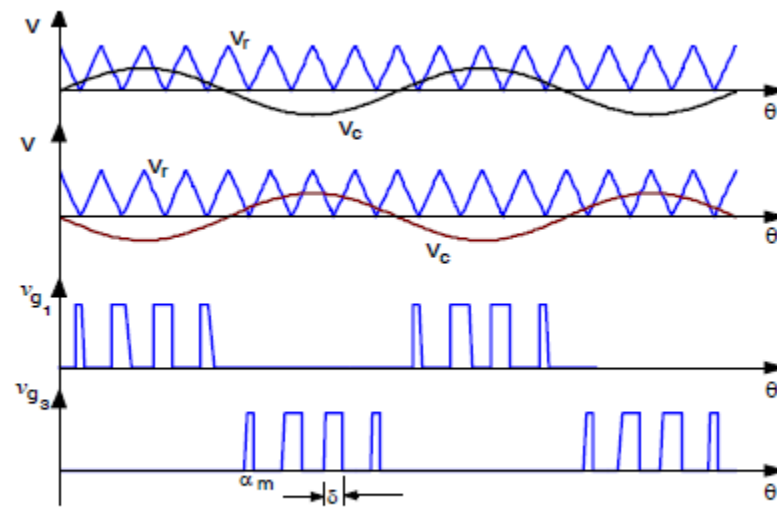


Fig. 4.44 Sinusoidal pulse-width modulation control

14. What are the main advantages of series connection of three phase full converter from the load and supply sides?

Supply side :

- 1- AC line in primary side is near from pure sinusoidal waveform
- 2- Fifth and seventh harmonics are effectively eliminated in the AC side
- 3- Reduction in harmonics reduces the cost of harmonic filter

Load Side

- 1- DC voltage ripple is reduced , where sixth and eighth harmonics are eliminated

Application : High voltage DC link (HVDCL)

1- Discuss Briefly a method to improve the PF

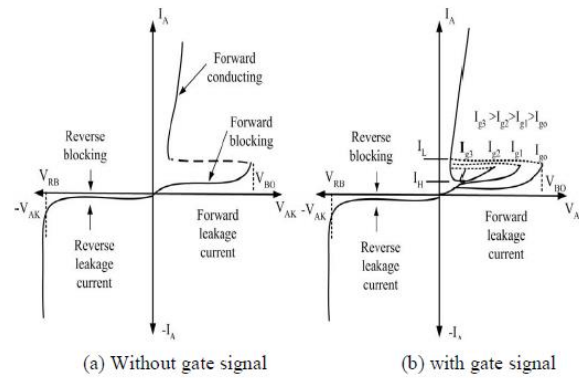
اشرح

Extinction angle or symmetrical angle control or PWM or SPWM

الاسهل extinction angle

2- Static and dynamic ch/c of thyristor and define :delay and rise time

a) Static ch/c



b) Dynamic ch/c

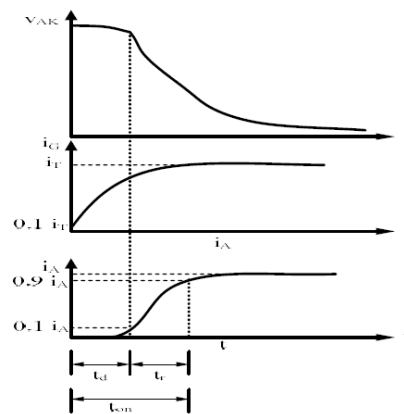


Fig. 2.27 Thyristor Dynamic Characteristics

The delay time is the time interval required from the gate current of 0.1 of its rated value to the anode current reaches to 0.1 of its rated value.

☐ **The rise time** is the time required to increase the anode current from 0.1 to 0.9 rated value.

☐ **The on time** is summation of both delay and rise time.

3- Compare – general purpose and fast recover and shottkey

	General Purpose diode	Fast recovery diode	Shottkey diode
Frequency range	<i>less 1kHz</i>		
Recovery time	<i>25μsec</i>	<i>Less 5μsec</i>	Only tb not di/dt
Cost	low	High (diffusion controlled by platinum or gold)	
Voltage and current	high	50 : 3kV 1A : hunderds A	Low vdrop (: 100,200V more leakage current (1 : 300 A)
application	Rectifier circuit	Rectifier and inverter	

4- What are the main advantages of series connection of three phase full converter from the load and supply sides?

Supply side :

- 4- AC line in primary side is near from pure sinsuidal waveform
- 5- Fifth and seventh harmonics are effectively eliminated in the AC side
- 6- Reduction in harmonics reduces the cost of harmonic filter

Load Side

- 2- DC voltage ripple is reduced , where sixth and eighth harmonics are eliminated

Application : High voltage DC link (HVDCL)

8)

$$\frac{V_m}{2\pi} (1 + \cos \alpha) = E + IR$$
$$I = \frac{\frac{V_m}{2\pi} (1 + \cos 45) - E}{R} = 4.466 \text{ A}$$

9) Half wave VS three phase rectifier

Higher Vdc Value

less Ripple factor means less ac component in the output as for half wave RF = 1.2114 but for three phase rectifier it is 0.0411

Also better Transformer utilization factor because in half wave the secondary winding of the transformer saturates because of the presence of a unidirectional current as current flows only on the positive cycle

So in sum up the three phase is much better in efficiency as 99.83 % than the half wave 40.5